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B.E./B.Tech. (FULL TIME) DEGREE END SEMESTER EXAMINATIONS, APRIL/MAY 2012

(COMMON TO ALL BRANCHES)

FIRST SEMESTER

MA 171 – MATHEMATICS I

(REGULATIONS 2004)

Time : 3 Hours

Max. Marks : 100

**Answer ALL Questions**

**Part – A ( 10 x 2 = 20 Marks)**

1. Using the properties of eigen values, find the eigen values of  $\text{Adj}(A)$  if the matrix  $A_{3 \times 3}$  has two eigen values 1, 1 and  $\det(A)=4$ .
2. Give the quadratic form corresponding to the symmetric matrix  $\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$ .
3. Find the equation of the sphere which has  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  as the extremities of a diameter.
4. Show that the general equation of the cone of the second degree which passes through the axes is of the form  $fyz+gzx+hxy=0$ .
5. Test for convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ .
6. State the D'Alembert 's ratio test for convergence of a positive term series.
7. State the chain rule for jacobians involving functions of two variables.
8. Using Euler's theorem, find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  where  $\log u = \frac{(x^3 + y^3)}{(3x + 4y)}$ .
9. Solve  $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 4y = 2x^2$  by the method of undetermined coefficients.
10. Find the particular integral of  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$ .

**Part – B ( 5 x 16 = 80 Marks)**

11. (a). (i). Find the equation of the sphere through the points ( 0,0,0), (0,1,-1), (-1,2,0) and (1,2,3). Locate its centre and find the radius. (8 Marks)

- (ii). Find the equation of the circular cylinder having for its base the circle  $x^2 + y^2 + z^2 = 9$  and  $x-y+z=3$ . (8 Marks)

12. (a). (i). Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ . (8 Marks)

- (ii). Verify Cayley-Hamilton theorem for the matrix  $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$  and find its inverse. Also express  $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$  as a linear polynomial in A. (8 Marks)

**(OR)**

- (b). Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$  to the canonical form. Also specify the matrix of transformation and its nature. (16 Marks)

13. (a). (i) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$ . (8 Marks)

- (ii) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ . (8 Marks)

**(OR)**

- (b). (i). Discuss the convergence of the series  $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots \infty$  (8 Marks)

- (ii). State the values of x for which the following series converge.

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \infty. \quad (8 \text{ Marks})$$

14. (a). (i). If  $v = (x^2 + y^2 + z^2)^{-1/2}$ , find  $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$ . (8 Marks)

(ii). Expand  $e^x \log(1+y)$  in powers of  $x$  and  $y$  upto second degree. (8 Marks)

(OR)

(b). (i). Discuss the maxima and minima of  $f(x, y) = x^3 y^2 (1 - x - y)$ . (8 Marks)

(ii). Find the maximum and minimum distances of the point  $(3, 4, 12)$  from the sphere  $x^2 + y^2 + z^2 = 1$ . (8 Marks)

15. (a). (i). Solve by the method of variation of parameter  $y'' - 2y' + y = e^x \log(x)$ . (8 Marks)

(ii). Solve  $x^2 \frac{d^2 x}{dy^2} - x \frac{dy}{dx} + y = \log(x)$  (8 Marks)

(OR)

(b). The small oscillations of a certain system with two degrees of freedom are given by the equations

$$D^2 x + 3x - 2y = 0$$

$$D^2 x + D^2 y - 3x + 5y = 0$$

where  $D = \frac{d}{dt}$ . If  $x=0, y=0, Dx = 3, Dy = 2$  when  $t=0$ , find  $x$  and  $y$  when  $t = 1/2$ . (16 Marks)

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