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Anna University, Chennai - 600 025
B.E / B.Tech (Full Time) Degree End Semester Examinations - May 2012
II Semester B.E /B.Tech - Common to All Branches
MA181 - Mathematics II- Regulation 2004

Duration: 3 Hours — Total marks= 100

Part A — (10 x 2 = 20 Marks)

1. Change the order of integration for the following integral $\int_0^1 \int_{x^2}^{2-x} f(x,y)dydx$.
2. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyz \ dxdydz$.
3. Find the directional derivative of $\phi = x + y + z$ at the point $(1, -2, 1)$ in the direction of the vector $\widehat{\mathbf{i}} + \widehat{\mathbf{j}} + \widehat{\mathbf{k}}$.
4. If $\overline{\mathbf{F}}$ is a conservative vector field, then what is the line integral $\int_C \overline{\mathbf{F}} \cdot d\overline{\mathbf{r}}$, where C is any path in the space joining two distinct points A and B?
5. Verify whether or not $f(z) = e^z$ is analytic.
6. If $f(z)$ is analytic and Real part of $f(z)$ is constant, then show that $f(z)$ is constant.
7. Find the fixed point of the mapping $w = \dfrac{z-2}{z+3}$.
8. Evaluate $\int_C \dfrac{2z}{(z-1)}dz$ using Cauchy's integral formula, where C is the circle $|z| = 2$.
9. Identify and classify the singularity of the function $f(z) = ze^{1/z}$.
10. Find the Laplace transform of $f(t) = t\sinh t$.
11. Find the inverse Laplace transform of $\dfrac{1}{(s+1)^2}$.

Part B — (5 X 16=80 Marks)

11.a(i) Solve $y'' - 6y' + 9y = t^2e^{3t}, y(0) = 2, y'(0) = 6$ by Laplace transform method. (8 Marks)

(ii) Using convolution theorem find the inverse Laplace transform of $\dfrac{1}{(s^2+4)^2}$. (8 Marks)

12.a(i) Find, using double integration the area enclosed by the planar curves $y^2 = 4x$ and $x^2 = 4y$. (8 Marks)

(ii) Find, by triple integral, the volume of the tetrahedron bounded by coordinate planes and the plane $x + y + z = 1$. (8 Marks)

(OR)

12.b(i) Evaluate, through the change of variables, the double integral $\int\int_R (x-y)^2 e^{(x+y)} dA$, where R is the square with vertices $(1,0),(2,1),(1,2)$ and $(0,1)$ using the transformation $u = x+y$ and $v = x-y$.

(8 Marks)

(ii) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.

(8 Marks)

13.a(i) Verify Green's theorem for $\oint_C [(2x-y)dx + (x+y)dy]$, where C is the boundary of the circle $x^2+y^2=4$ in the xy-plane. (16 Marks)

(OR)

13.b(i) Verify Gauss divergence theorem for $\int\int\int_E div\mathbf{F}\, dV$ for the vector valued function $\bar{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, where E is the cube bounded by the planes $x=0, x=1, y=0, y=1, z=0$ and $z=1$.

(16 Marks)

14.a(i) Show that $u(x,y) = x^3 - 3xy^2 + 3x^2 - 3y^2$ is a harmonic function. Find $v(x,y)$ such that $f(z) = u(x,y) + iv(x,y)$ is analytic. Express $f(z)$ in terms of z. (10 Marks)

(ii) Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into the points $w = i, -1, -i$ respectively. (6 Marks)

(OR)

14.b(i) Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function. Show that the one parameter families of curves $u(x,y) = c_1$ and $v(x,y) = c_2$ cut orthogonally each other. (6 Marks)

(ii) Find the image of (i) the circle $|z-3i| = 1$ and (ii) the line $y = 2x$ under the map $w = 1/z$.

(10 Marks)

15.a(i) Find the Laurent's series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ in valid in the region (i) $1 < |z| < 2$ (ii) $|z| > 2$ (8 Marks)

(ii) Using Contour integration, evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx$. (8 Marks)

(OR)

15.b(i) Using Contour integration, evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{(5-4\cos\theta)}$. (8 Marks)

(ii) Using Cauchy's residue theorem, evaluate $\displaystyle\int_C \frac{(z-1)}{(z+1)(z-2)} dz$ where $C : |z-i| = 4$

(8 Marks)

–Paper Ends–