

Regn. No:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Anna University, Chennai 600 025
B.E / B.Tech (Full Time) Degree End Semester Examinations - MAY 2012
II Semester B.E /B.Tech - Common to All Branches
MA9161 - Mathematics II- (Regulation 2008)

Duration: 3 Hours

Total marks= 100
(10 x 2 = 20 Marks)Part A

1. Find the particular integral of $(D^2 - 2D)y = e^x \cos x$.
2. Guess the trial solution of the particular integral for the differential equation $y'' + y = \sin x$ using method of undetermined coefficients.
3. Find the directional derivative of $\phi = x^2y^2z^2$ at the point $(1, 1, 1)$ in the direction of the vector $2\hat{i} + \hat{j} + 2\hat{k}$.
4. If $\vec{F} = y^2\hat{i} + 2(xy + z)\hat{j} + 2y\hat{k}$, state whether or not $\int_C \vec{F} \cdot d\vec{r}$ is path dependent.
5. If $f(z)$ is analytic and $\overline{f(z)}$ is also analytic, then show that $f(z)$ is constant.
6. Find the image of $|z - 2i| = 2$ under the mapping $w = z + 3 - 2i$.
7. Evaluate $\int_C \frac{5z^2 + 6z + 1000}{(z - 1)^2} dz$, where C is the circle $|z| = 5/2$.
8. Identify and classify the singularity of the function $f(z) = (\cos z - 1)/z$.
9. Find the Laplace transform of $f(t) = t \sin t$.
10. Using Initial Value theorem, find $\lim_{t \rightarrow 0} f(t)$, given that $L\{f(t)\} = \frac{1}{s(s+2)}$.

Part B

(5 X 16=80 Marks)

- 11.a(i) Using method of variation of parameters solve the following differential equation
 $y'' - 2y' + y = xe^x$. (8 Marks)
- 11.a(ii) Solve $(x^2D^2 - 3xD + 4)y = x \log x$. (8 Marks)
- 12.a(i) Verify Green's theorem for $\oint_C \{(xy)dx + x^2dy\}$, where C is the boundary of the square in the xy - plane formed by $x = 0, y = 0, x = 1$ and $y = 1$. (8 Marks)
- (ii) Verify Stoke's theorem for the vector field $\vec{F} = -y\hat{i} + 2yz\hat{j} + y^2\hat{k}$, where S is hemisphere $x^2 + y^2 + z^2 = 1$ and C is its boundary on the xy - plane. (8 Marks)
(OR)
- 12.b(i) Verify Gauss divergence theorem for the vector function $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$, where E is the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. (10 Marks)

(ii) Verify that $\vec{F} = y^2\hat{i} + 2xy\hat{j} + 2z\hat{k}$ is irrotational; further find also its corresponding scalar potential.

(6 Marks)

13.a(i) Find the analytic function $f(z) = u(x, y) + iv(x, y)$, given that $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$.
(8 Marks)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$ respectively. Find also the pre-image of $|w| = 1$ under this bilinear transformation. (8 Marks)

(OR)

13.b(i) Under the mapping $w = z^2$, find the image (in w -plane) of the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ in z -plane. Sketch vertices and boundary lines of original triangular object and the corresponding image object. (8 Marks)

(ii) Find the image of the circle $|z - i| = 1$ and the line $x + y = 1$ under the map $w = 1/z$.
(8 Marks)

14.a(i) Find the Laurent's series expansion of $f(z) = \frac{1}{z^2 - 3z + 2}$ valid in the region (i) $1 < |z| < 2$
(ii) $|z| > 2$. (8 Marks)

(ii) Using Contour integration on unit circle, evaluate $\int_0^{2\pi} \frac{d\theta}{(5 + 4 \cos \theta)}$.
(8 Marks)

(OR)

14.b(i) Using Contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 9)(x^2 + 4)} dx$. (8 Marks)

(ii) Using Cauchy's residue theorem, evaluate $\int_C \frac{dz}{(z^2 + 4)^2}$, where $C : |z - i| = 2$. (8 Marks)

15.a(i) Solve $y'' - 2y' + y = te^t, y(0) = 1, y'(0) = 1$ by Laplace transform method. (8 Marks)

(ii) Using convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2 + 1)^2}$. (8 Marks)

(OR)

15.b(i) Find the Laplace transform of the periodic function defined on the interval $0 \leq t \leq 1$ by

$$f(t) = \begin{cases} -1, & 0 \leq t < 1/2 \\ 1, & 1/2 \leq t < 1 \end{cases} \quad \text{and } f(t+1) = f(t). \quad (8 \text{ Marks})$$

(ii) Find the inverse Laplace transform of $\frac{s + 1}{(s + 1)(s^2 + 2s + 2)}$. (8 Marks)

–Paper Ends–