Reg. No. ☐☐☐☐☐☐☐☐☐☐☐☐

B.E. / B.Tech. END SEMESTER EXAMINATIONS, APRIL 2012

I SEMESTER

MA 9111 MATHEMATICS-I
(Common to all Branches)

Time: 3 Hours **Answer All the Questions** **Max. Marks: 100**

Part A **($10 \times 2 = 20$)**

1. Find the eigenvalues of $2A^2$ given that $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$.

2. Determine the characteristic equation satisfied by the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

3. When is a series said to be conditionally convergent? Give an example.

4. State the necessary condition for the convergence of a positive term series.

5. State the Euler's theorem for homogenous functions of degree 'n' in three independent variables.

6. Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ given that $u = x^2 - 2y$; $v = x + y$.

7. Evaluate the improper integral $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

8. Test the convergence of the improper integral $\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2}$.

9. Sketch the region of integration of the integral $\int_{1}^{2}\int_{-x}^{x} f(x, y)\,dy\,dx$.

10. Change the order of integration in $\int_{0}^{a}\int_{x/a}^{\sqrt{x/a}} g(x, y)\,dy\,dx$.

Part B **(5 × 16 = 80)**

11. (i) Reduce the following quadratic form to canonical form by an orthogonal transformation: $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$. (10 marks)

(ii) Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$. (6 marks)

12a. (i) Define the interval of convergence of a power series. Find the same for the series $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots$. (8 marks)

(ii) State Leibnitz's rule and use it to test the convergence of the following alternating series: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n+1}$. (8 marks)

(OR)

b. (i) State the D'Alembert's ratio test. Using it, test the convergence of the following series : $\sum \frac{(2n)!}{(n!)^2}$. (8 marks)

(ii) Define absolute convergence of a series. Test whether the following series is absolutely convergent or not: $1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \frac{1}{7^2} - \frac{1}{8^2} + \cdots$ (8 marks)

13a. (i) If $u = f(2x-3y, 3y-4z, 4z-2x)$, then find the value of $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$.

(8 marks)

(ii) Using Taylor's series expand $\sin x \sin y$ in powers of x and y as far as the terms of third degree. (8 marks)

(OR)

b. (i) Discuss the maxima and minima and obtain the extreme values of the function: $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. (10 marks)

(ii) If $\sin u = \frac{x^2 y^2}{x+y}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. (6 marks)

14a. (i) Prove that $\int_0^\infty \sqrt{x} e^{-x^2} dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$. (8 marks)

(ii) Show that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$, where n is a positive integer and $m > -1$. (8 marks)

(OR)

b. (i) Using Leibnitz's rule, evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2} dx$. (8 marks)

(ii) Show that $\int_0^t erf_c(ax) dx = t\, erf_c(at) - \frac{e^{-a^2 t^2}}{a\sqrt{\pi}} + \frac{1}{a\sqrt{\pi}}$. (8 marks)

15a. (i) Find by double integration, the area of the lemniscate $r^2 = a^2 \cos 2\theta$.

(8 marks)

(ii) By changing into spherical polar coordinates, evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. (8 marks)

(OR)

b. (i) Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} + \left(\frac{z}{a}\right)^{2/3} = 1$. (8 marks)

(ii) Evaluate $\iint r^3 dr d\theta$ over the area between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. (8 marks)