Roll No.										

B. E./B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, APRIL/MAY 2012

Common to all Branches

THIRD SEMESTER

MA 271 MATHEMATICS III.

(REGULATION 2004)

Time: 3 Hours. Answer All questions **Max. Mark: 100**

PART A (10 X 2 = 20)

1. Form a partial differential equation by eliminating the arbitrary function from $z = f(x+2y)$.
2. Obtain the complete integral of the partial differential equation $2p + q = 3$.
3. State Dirichlet's conditions on Fourier series expansion.
4. In the Fourier series expansion of $f(x) = (x^2 - 3)$ in $(-\pi, \pi)$, what is the value of b_2.
5. If a string of length 10 cm long is initially at rest in its equilibrium position and each of its points is given a velocity v such that $v = x(10 - x), \quad 0 < x < 10$ to vibrate the string. State the differential equation and conditions related to this problem.
6. A rod AB of length 20 cm has the ends A and B kept at temperature $30^\circ C$ and $110^\circ C$, respectively. Find the steady state temperature distribution on the rod.
7. State Fourier integral theorem.
8. Define Fourier sine transform of $f(x)$ and its inverse Fourier sine transform.
9. Define Z-transform of the sequence $\{f(n)\}$.
10. Obtain a difference equation by eliminating arbitrary constants from $y_n = 3^n + a$.

PART – B (5 X 16 = 80)

11. (i) Find the integral surface of $xp - yq = z$ which passes through the circle $x^2 + y^2 = 1, \; z = 1$. **(8)**

(ii) Solve: $(D^2 + 3DD' + 2D'^2)z = e^{x-2y} + \sin(2x+y)$. **(8)**

12. a) i) Obtain the half range sine series for the function $f(x) = x$ in the interval $0 < x < \ell$. Hence show that $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - - -$ and $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + - - -$.

ii) Find the complex form of the Fourier series of the function $f(x) = e^{-x}$ when $-1 < x < 1$ and $f(x+2) = f(x)$. **(10+6)**

OR

b) i) Find the Fourier series for $f(x)=\begin{cases} x & in\ 0\le x\le \pi \\ 2\pi - x & in\ \pi\le x\le 2\pi \end{cases}$. **(8)**

ii) Obtain a Fourier series upto the second harmonic from the data

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(8)

13.a) A string is stretched tightly between $x=0$ and $x=20$ and is fastened at both ends. At time $t=0$, the string is given a shape defined by $f(x)=x(20-x)$ and then released. Find the displacement of any point x of the string at any time t. **(16)**

OR

b) A rectangular plate with insulated surfaces is π cm wide and so long compared to its width that it may be considered infinite in length with out introducing an appreciable error. If one short edge $y=0$ is kept at $20°C$ while the two long edges $x=0$ and $x=\pi$, as well as the other short edge, are kept at $0°C$, find the steady state temperature at any point on the plate. **(16)**

14.a) Find the Fourier Transform of $f(x)$ given by $f(x)=\begin{cases} 1 & for|x|<a \\ 0 & for|x|>a>0 \end{cases}$. By inverting it, evaluate $\int_0^\infty \frac{\sin as\ \cos sx}{s}ds$ and deduce that $\int_0^\infty \frac{\sin t}{t}dt=\frac{\pi}{2}$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$. **(16)**

OR

b) Find the Fourier sine and cosine transform of $f(x)=e^{-ax}$, $a>0$ and deduce that $\int_0^\infty \frac{\cos sx}{a^2+s^2}ds=\frac{\pi}{2a}e^{-ax}$ and $\int_0^\infty \frac{s\sin sx}{a^2+s^2}ds=\frac{\pi}{2}e^{-ax}$. **(16)**

15.a)(i) Find the Z-transforms of $\{n\}$, $\{na^n\}$ and $\{n^2a^n\}$ **(2+3+3)**

(ii) Find the inverse Z-transform of $\frac{z}{(z-1)^2(z+1)}$. **(8)**

OR

b)(i) Using convolution theorem find the inverse Z-transform of $\frac{z^2}{(z-4)(z+3)}$. **(8)**

(ii) Solve, by using Z-transform, $y_{n+2} + y_n = 2$ given that $y_0 = y_1 = 0$. **(8)**

&&&&&&&