Regn. No: | | | | | | | | | |

B.E / B.Tech (Full Time) Degree End Semester Examinations - April / May 2014
Electronic and Communication Engineering Branch
IV Semester B.E (ECE)
MA8401 Linear Algebra and Numerical Methods
(Regulation 2012)

Duration: 3 Hours — Total marks= 100

Part A — (10 x 2 = 20 Marks)

1. Is the set of all points on the line $x + y = 1$ in the xy-plane, a vector subspace of $\mathbb{R}^2$ with respect to usual vector addition and scalar multiplication?
2. What is the linear span $L(S)$ of the set $S = \{\bar{e}_1, \bar{e}_2, (\bar{e}_1 + \bar{e}_2)/2\}$ where $\bar{e}_1 = (1, 0, 0)$ and $\bar{e}_2 = (0, 1, 0)$ are elements in $\mathbb{R}^3$?
3. Write down the matrix form of the linear transformation that is described by a rotation about an angle θ in the counter clockwise direction in xy-plane.
4. State TRUE or FALSE with proper justification: "Linearly independent vectors are always orthogonal in an inner product space".
5. Prove that orthogonal complement of any set S of an inner product space V is a subspace of V.
6. Show that similar matrices have same eigenvalues.
7. Construct a LU decomposition of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.
8. Explain row pivoting in connection with Gauss Elimination Method.
9. Define generalized inverse for a non-square matrix.
10. Show that singular values for a symmetric matrix coincide with its eigenvalues.

Part B — (5 X 16=80 Marks)

11. (i) Solve the following system by Gauss elimination method:

$$\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 2z &= 18 \\ x + 4y + 9z &= 16. \end{aligned}$$

(8 Marks)

(ii) Write down the Gauss-Seidel iteration scheme for the following system. Then solve the system by the same method for three iterations starting with the initial vector $(0, 0, 0)^T$.

$$\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25. \end{aligned}$$

(8 Marks)

12.a(i) Let V be the set of of all $2X2$ matrices with real entries. Show that V is a vector space over $\mathbb{R}$ with respect to *usual matrix addition done entry wise* and *usual scalar multiplication done entry wise.* Verify all the conditions of a vector space. (8 Marks)

(ii) Let $\mathcal{B} = \{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n\}$ be a subset of a vector space V. Then $\mathcal{B}$ is a basis if and only if each $\bar{v} \in V$ can uniquely be expressed as a linear combination of vectors of $\mathcal{B}$. (8 Marks)

(OR)

12.b(i) Determine whether or not the set $\mathcal{S} = \{1 + 2x + x^2, 3 + x^2, x + x^2\}$ forms a basis for $\mathbb{P}_2(\mathbb{R})$. (8 Marks)

(ii) Prove that the linear span $L(S)$ of a subset S of a vector space $(V, +, .)$ over $\mathbb{R}$ is a vector subspace of V. Further, show that $L(S)$ is the smallest subspace that contains S. (8 Marks)

13.a(i) Using Gram-Schmidt orthogonalization process construct an orthogonal set from the given set $S = \{(1,0,1), (0,1,1), (1,3,3)\}$ of $\mathbb{R}^3$. Also find the Fourier coefficient of the vector $(1,1,2)$ with respect to the resultant orthogonal vectors. (8 Marks)

(ii) Let $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_3(\mathbb{R})$ be defined by

$$T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt.$$

Find bases for $N(T)$ and $R(T)$ and hence verify the dimension theorem. Is T one-to-one? Is T onto? Justify your answer. (8 Marks)

(OR)

13.b(i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x,y) = (2x - y, 3x + 4y, x)$. Compute the matrix of the transformation with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. Find $N(T)$ and $R(T)$. Is T one-to-one, onto? Justify your answer. (8 Marks)

(ii) Let V and W be vector spaces and let $T : V \to W$ be a linear transformation. If V is finite-dimensional then prove that

$$nullity(T) + rank(T) = dimension(V).$$

(8 Marks)

14.a(i) For the linear operator $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ defined as $T(f(x)) = f(x) + xf'(x) + f''(x)$, find the eigenvalues of T and an ordered basis $\mathcal{B}$ for $P_2(\mathbb{R})$ such that the matrix of the given transformation with respect to the new resultant basis $\mathcal{B}$ is a diagonal matrix. (8 Marks)

(ii) Using Least square approximation determine the best linear fit for the data: $\{(1,2), (2,3), (3,5), (4,7)\}$.

(8 Marks)

(OR)

14.b(i) Solve the system of differential equations using diagonalization and discuss its stability:

$$\begin{aligned} x'(t) &= 5x(t) + 4y(t) \\ y'(t) &= x(t) + 2y(t). \end{aligned}$$

(8 Marks)

(ii) Let V be an inner product space over $\mathbb{R}$. For all $\bar{x}, \bar{y} \in V$ prove the following:
(1) Cauchy-Schwarz inequality $| < \bar{x}, \bar{y} > | \leq \|\bar{x}\| \; \|\bar{y}\|$ and
(2) Triangle inequality $\|\bar{x} + \bar{y}\| \leq \|\bar{x}\| + \|\bar{y}\|$. (8 Marks)

15.a(i) Obtain by power method the numerically largest eigenvalue and its corresponding eigenvector for the matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{pmatrix}$ starting with the vector $(1, 1, 0)^T$ for three iterations.

(8 Marks)

(ii) Construct a QR decomposition for the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. (8 Marks)

(OR)

15.b(i) Using the Jacobi rotation method, find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & 2 \end{pmatrix}$. (8 Marks)

(ii) Construct a singular value decomposition for the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & -1 \end{pmatrix}$. (8 Marks)

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