Regn. No: | | | | | | | | | | |

B.E/B.Tech(Full Time) DEGREE END SEMESTER EXAMINATIONS, MAY 2013
COMMON TO ALL BRANCHES
FIRST SEMESTER
MA8151 MATHEMATICS-I
(REGULATION 2012)

Time: 3 Hours

Maximum Marks:100

Answer ALL Questions

Part- A(10x2=20 Marks)

1. If λ is an eigen value of A, show that $\lambda + 2$ is an eigen value of $A + 2I$.
2. Determine the nature of the quadratic form $x_1^2 + 2x_2^2 + 3x_3^2$.
3. Examine the convergence of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \cdots$.
4. Test whether the following series is convergent: $1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{9}} - \cdots$.
5. Is the function $f(x, y, z) = \frac{x^3 + 2y^2 z}{x + y + z}$ homogeneous? If so, what is its degree?
6. If $x = \rho\cos(\phi), y = \rho\sin(\phi), z = z$, find the Jacobian $\frac{\partial(x, y, z)}{\partial(\rho, \phi, z)}$.
7. Check whether the improper integral $\int_{-3}^{3} \frac{1}{x^2} dx$ converges.
8. Show that $erf(x) + erf(-x) = 0$.
9. Evaluate $\int_0^1 \int_0^x e^{x+y} \; dydx$.
10. Sketch the region of integration in the integral $\int_0^\infty \int_x^\infty f(x, y) dydx$.

Part- B(5x16=80 Marks)

11a. i. Diagonalize the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by means of an orthogonal transformation. (10)

ii. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence express A^{-1} as a linear combination of A and I. (6)

12.a. i. Test the series $\sum_{n=1}^{\infty}(\sqrt{n^2+1}-n)$. (8)

ii. For what values of x the following series converges: $x + 2x^2 + 3x^3 + 4x^4 + \cdots$. (8)

OR

b. i. Discuss the convergence of the series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \frac{5!}{5^5} + \cdots$. (8)

ii. Test the convergence of the series $1 + \frac{1}{x} + \frac{1}{2x^2} + \frac{1}{3x^3} + \cdots$. (8)

13.a. i. Expand $e^x \log(1+y)$ in powers of x and y upto second degree. (6)

ii. Discuss the maxima and minima of $f(x,y) = x^2y^2(1-x-y)$. (10)

OR

b. i. A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. (8)

ii. If z is a homogeneous function of degree n in x and y, then show that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$ (8)

14. a i. Using differentiation under the integral sign, prove that $\int_0^{\infty} \frac{e^{-x}}{x}(1-e^{-ax})dx = \log(1+a)$, where $a > -1$. (8)

ii. Prove that

$$\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}dx = \frac{\pi}{4\sqrt{2}}$$

(8)

OR

b. i. Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of gamma functions and hence evaluate $\int_0^1 \frac{dx}{\sqrt{(1-x^n)}}$. (8)

ii. Prove that $\beta(m, \frac{1}{2}) = 2^{2m-1}\beta(m,m)$ and hence show that

$\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. (8)

15.a. i. Find the area lying inside the cardioid $r = a(1 - cos(\theta))$ and outside the circle $r = a$. (8)

ii. Calculate the volume of the solid bounded by the planes $x = 0, y = 0, x + y + z = a$ and $z = 0$. (8)

OR

b. i. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2}dx = \sqrt{\frac{\pi}{4}}$. (8)

ii. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (8)