Roll No | | | | | | | | | | |

B.E. / B.TECH. (Full Time) DEGREE END SEMESTER EXAMINATIONS, NOVEMBER 2013

(2)

COMMON TO ALL BRANCHES

First Semester

MA 8151 Mathematics - I

(Regulation 2012)

Time : 3 Hours | Answer **ALL** Questions | **Max. Marks:** 100

PART-A (10 x 2 = 20 Marks)

1. The sum and product of the eigenvalues of a 2x2 matrix A are 2 and -3 respectively. Compute the eigenvalues of A and identify the matrix A which is non-diagonal.
2. Find the index and signature of the quadratic form $x^2 + 2y^2 - 3z^2$.
3. State the necessary and sufficient conditions for the convergence of a sequence $\{a_n\}$.
4. State the interval of convergence of exponential series e^x and logarithmic series $\log(1+x)$.
5. Find dy/dx when $x^2 + y^2 = xy$.
6. Are $u(x,y) = x/y$ and $v(x,y) = (x+y)/(x-y)$ functionally dependent ? If it so, find the functional relation between them.
7. Evaluate the improper integral $\int_1^\infty x e^{-x^2} dx$.
8. Show that the sum of error function of x and complementary error function of x is unity.
9. Sketch the region of integration of the integral $\int_0^{2a} \int_{\sqrt{(2ax-x^2)}}^{\sqrt{(2ax)}} f(x,y)dy\,dx$ and change the order of integration.

10. Evaluate the triple integral $\int_0^1 \int_0^y \int_0^{x+y} dz\,dx\,dy$.

Part – B (5 x 16 = 80 Marks)

11.(i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$ by changing to polar co-ordinates and hence find the value of $\int_0^\infty e^{-x^2} dx$. (8)

11.(ii) Find, by using triple integrals, the volume of the sphere $x^2 + y^2 + z^2 = a^2$. (8)

12a.(i) Find the eigenvalues and the corresponding eigenvectors of a matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}.$$ (8)

12a.(ii) The eigenvectors of a 3x3 real symmetric matrix A corresponding to eigenvalues -1, 1 and 4 are $(0\ \ 1\ \ 1)^T$, $(2\ \ -1\ \ 1)^T$ and $(1\ \ 1\ \ -1)^T$ respectively. Find the matrix A by using suitable transformation. (8)

(OR)

12b.(i) Show that $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ satisfies its own characteristic equation and hence find A^{-2}. (7)

12b.(ii) Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6xz$ to the canonical form by orthogonal transformation. (9)

13a.(i) Examine the character of the series $\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-\frac{x^4}{1+x^4}+.....\text{to } \infty$, where $0 < x < 1$. (8)

13a.(ii) Test for the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt[3]{(n^3+1)}-n\right)$, using comparison test or any one of the test. (8)

(OR)

13b.(i) Test for the convergence of the series $1+\frac{(1+\alpha)}{(1+\beta)}+\frac{(1+\alpha)(1+2\alpha)}{(1+\beta)(1+2\beta)}$ $+\frac{(1+\alpha)(1+2\alpha)(1+3\alpha)}{(1+\beta)(1+2\beta)(1+3\beta)}+.....\text{to } \infty$, where $\alpha,\beta > 0$, by D'Alembert's ratio test. (8)

13b.(ii) Test the series $\frac{2}{3}-\frac{3}{4}\frac{1}{2}+\frac{4}{5}\frac{1}{3}-\frac{5}{6}\frac{1}{4}+.....\text{to } \infty$ for absolute convergence and conditional convergence. (8)

14a.(i) If $u = x^2\tan^{-1}(y/x)-y^2\tan^{-1}(x/y)$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2}$ $+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$, using Euler's theorem, and also show that $\frac{\partial^2 u}{\partial x\,\partial y}$ $=\frac{x^2-y^2}{x^2+y^2}$. (8)

14a.(ii) Expand $e^x \sin y$ in powers of $(x+1)$ and $(y-(\pi/4))$ up to second degree terms by using Taylor's theorem. (8)

(OR)

14b.(i) If $z=f(x,y)$, where $x=e^u\cos v$ and $y=e^u\sin v$, then show that

$$x\frac{\partial z}{\partial v}+y\frac{\partial z}{\partial u}=e^{2u}\frac{\partial z}{\partial y}.$$ (8)

14b.(ii) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, using Lagrange's method. (8)

15a.(i) Evaluate $\int_0^{\infty}\frac{\tan^{-1}(\alpha x)}{x(1+x^2)}dx$, where $\alpha\geq 0$, using Leibnitz's rule for constant limits of integration. (8)

15a.(ii) Test for the convergence of the improper integral $\int_0^{\pi/2}\frac{\sin x}{x^{3/2}}dx$. (8)

(OR)

15b.(i) Evaluate $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of Gamma functions and hence find the value of $\int_0^1\frac{dx}{\sqrt{(1-x^n)}}$. (8)

15b.(ii) Evaluate Dirichlet's integral $\iiint_V x^{\ell-1}y^{m-1}z^{n-1}\,dx\,dy\,dz$ in terms of Gamma functions, where V is the region $x\geq 0, y\geq 0, z\geq 0$ and $x+y+z\leq 1$. (8)