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B.E. / B.Tech. END SEMESTER EXAMINATIONS (ARREAR)
DECEMBER 2012

I SEMESTER

5

MA 9111 MATHEMATICS-I
(Common to all Branches)

Time: 3 Hours | **Answer All the Questions** | **Max. Marks: 100**

Part A $(10 \times 2 = 20)$

1. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$, then find the eigenvalues of A^4 and A^{-1}.

2. State Cayley-Hamilton theorem.

3. Test for convergence of the series $6 - 10 + 4 + 6 - 10 + 4 + 6 - 10 + 4 + \cdots$.

4. State the Leibnitz's rule for testing the convergence of an alternating series.

5. Find the first order derivatives of $z = x^y$.

6. Find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$ given that $x = uv,\ y = \dfrac{u}{v}$.

7. Classify the improper integral $\int_0^{\pi/2} \tan x dx$ and test for convergence.

8. Compute $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$.

9. Give the area as a double integral of the region lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$.

10. Change the order of integration in the integral $\int_0^1 \int_{x^2}^{2-x} f(x,y)\,dy\,dx$.

Part B **(5 × 16 = 80)**

11 (i) Using Cayley Hamilton theorem, find the inverse of the matrix A, where

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}.$$ (8)

(ii) Reduce the quadratic form $2xy + 2yz + 2zx$ to a canonical form by orthogonal reduction.

(8)

12a. (i) Test for conditional convergence of the following series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$. (8)

(ii) Using D'Alembert's ratio test, check for the nature of the series

$$\frac{a+1}{b+1} + \frac{(a+1)(2a+1)}{(b+1)(2b+1)} + \frac{(a+1)(2a+1)(3a+1)}{(b+1)(2b+1)(3b+1)} + \cdots$$ (8)

(OR)

12 (i) Find the interval of convergence of the logarithmic series

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots \qquad (8)$$

(ii) Test for convergence of the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$, $(p>0)$. (8)

13a. (i) Expand the function $f(x,y)=e^x\log(1+y)$ in powers of x and y upto the third degree terms. (8)

(ii) Verify Euler's theorem for the function $z=x^n\log\left(\frac{x}{y}\right)$. (8)

(OR)

13b. (i) Find the shortest and the longest distance of the point (3,4,12) from the unit sphere whose centre is at the origin. (8)

(ii) If $z=z(x,y);\ x=e^u+e^{-v},\ y=e^{-u}-e^v$, prove that

$$\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}. \qquad (8)$$

14a. (i) Express the integral $\int_0^1 x^{3/2}\left(1-x^2\right)^{5/2}dx$ in terms of gamma functions.

(8)

(ii) Evaluate the improper integral $\int_{-1}^{1} \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$ and hence test for convergence. (8)

(OR)

14b. (i) Using Leibnitz's rule, evaluate $\int_0^1 x^m (\log x)^n\,dx$. (10)

(ii) Find $\frac{d}{dx}\left[erf(ax)\right]$. (6)

15a. (i) Find by triple integration, the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(8)

(ii) By changing to polar co-ordinates, evaluate $\int_0^{4a} \int_{y^2/4a}^{y} \frac{x^2 - y^2}{x^2 + y^2}\,dxdy$. (8)

(OR)

15b. (i) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (8)

(ii) Evaluate the triple integral $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\,dxdydz$. (8)