REG. NO

B.E. / B.TECH *(Full Time)* DEGREE END SEMESTER EXAMINATION, MARCH 2011

CIVIL ENGINEERING

THIRD SEMESTER

MA 9211 MATHEMATICS –III

(COMMON TO ALL BRANCHES)

(REGULATION 2008)

TIME: THREE HOURS **MAX.:100 MARKS**

Answer ALL Questions

PART-A (10 x 2 = 20 Marks)

1. Form the partial differential equation by eliminating the arbitrary function f from $z = f\left(x^2 + y^2\right)$.
2. Find the complete solution $\sqrt{p} + \sqrt{q} = \sqrt{x}$.
3. If $f(x)$ is an even function in $(-\pi, \pi)$ write down Euler's formula for finding the coefficients of the Fourier series.
4. If $f(x) = \begin{cases} x, & 0 \le x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$, then find the value of b_n.
5. What does a^2 represent in the PDE of a vibrating string $u_{tt} = a^2 u_{xx}$?
6. Write all possible solutions of one dimensional heat flow equation $u_t = a^2 u_{xx}$.
7. State Fourier integral theorem.
8. If $F(s)$ is the Fourier transform of $f(x)$, then find the the Fourier transform of $f(x-a)$.
9. State and prove the second shifting theorem in Z-transform.
10. If $Z[f(n)] = F(z)$, then, show that $Z[n f(n)] = -z \frac{d}{dz} F(z)$.

PART- B (5 x 16 = 80 Marks)

11. A rectangular plate with insulated surface is 10cm wide and so long. If the temperature at short edge y = 0 is $u(x,0) = 8\sin\left(\frac{\pi x}{10}\right), 0 < x < 10$, while two long edges x = 0 and x = 10 as well as the other short edges are kept $0^\circ C$, find the steady state temperature function u(x,y) at any point of the plate. (16)

12(a)(i) Solve $x(y-z)p + y(z-x)q = z(x-y)$. (8)

(ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(3x+2y)$. (8)

(OR)

(b)(i) Find the singular integral of $z = px + qy + 2\sqrt{pq}$. (8)

(ii) Find the complete integral of $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$. (8)

13(a)(i) Obtain Fourier series for f(x) of period 2ℓ and is defined as

$f(x) = \begin{cases} \ell - x, 0 < x \le \ell \\ 0 \quad , \ell \le x < 2\ell \end{cases}$ Hence deduce the sum $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$. (8)

(ii) Find the Fourier series as far as the second harmonic to represent the function given by the following data: (8)

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

(OR)

(b) (i) Find the half-range sine series for $f(x) = x(\pi - x)$ in the interval $(0,\pi)$ and deduce $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots$ (8)

(ii) Find the Fourier series of $f(x) = x^2$ in $-\pi < x < \pi$. Hence prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. (8)

14(a) Find the Fourier transform of $f(x) = \begin{cases} a - |x| & \text{for} \quad |x| < a \\ 0 & \text{for} \quad |x| > a > 0 \end{cases}$ and deduce the value of (i) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ (ii) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$. (16)

(OR)

(b)(i) Find the Fourier cosine transform of $f(x) = e^{-x}$. (4)

(ii) Find the Fourier sine transform of $f(x) = e^{-ax}$ and $g(x) = e^{-bx}$ for $a, b > 0$. Prove that

$$\int_0^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2(a+b)}, \text{ for } a, b > 0.$$ (12)

15(a)(i) If $U(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the values of u_2 and u_3. (8)

(ii) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms. (8)

(OR)

(b)(i) Form the difference equation by eliminating the constants a and b from $y_n = a.2^n + b(-2)^n$. (6)

(ii) Find the inverse Z-transform of $\frac{z^3 + 3z}{(z-1)^2(z^2+1)}$. (10)

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