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ANNA UNIVERSITY
B.E. (FULL TIME) DEGREE END SEMESTER EXAMS, APRIL 2011
SEMESTER IV – (REGULATIONS 2008)
MA 9263 - PROBABILITY AND RANDOM PROCESSES

(Common to Bio-medical and Electronics & Communications Engineering)

Time : 3 Hours **Answer ALL Questions** **Max. Marks : 100**

Part A **$(10 \times 2 = 20)$**

1) Find $E(X)$ for a random variable X whose probability mass function is given by $P(x) = \left(\frac{32}{63}\right)\frac{1}{2^x}$, for $x = 0,1,2,3,4$ and 5.

2) State and prove memoryless property of geometric distribution.

3) The equations of two regression lines obtained in a correlation analysis are as follows: $3x + 12y = 19$; $3y + 9x = 46$. Obtain the correlation coefficient.

4) Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} 2xy + \left(\frac{3}{2}\right)y^2, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}.$$ Find $P(X + Y < 1)$.

5) Prove autocorrelation function of a wide sense stationary process is even.

6) Find $E[X(t)]$, in the fair coin experiment, we define the process $\{X(t)\}$ as

$$X(t) = \begin{cases} \sin\pi t, \text{if head shows} \\ 2t, \text{if tail shows} \end{cases}.$$

7) A random process has autocorrelation function $R_{XX}(\tau) = \frac{16\tau^2+28}{\tau^2+1}$. Find the variance of the process.

8) Find the mean square value for the power spectral density function

$$S_{XX}(\omega) = \begin{cases} 0.01, 400\pi \le |\omega| \le 500\pi \\ 0, \; otherwise \end{cases}.$$

9) Find the power transfer function of a linear system whose impulse response is $h(t) = 2e^{-t} for\ t \ge 0$.

10) Assume that the input $X(t)$ to a linear time invariant system is white noise. What is the power spectral density of the output process $Y(t)$, if the system response is

$$H(\omega) = \begin{cases} 1, & \omega_1 < |\omega| < \omega_2 \\ 0, & \text{otherwise} \end{cases}.$$

Part B (5 × 16 = 80)

11) (i) In a large consignment of electric bulbs 10% are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs, (2) atmost there are three defective bulbs (3) exactly there are three defective bulbs. (8)

(ii) Find the moment generating function of exponential distribution and hence find the mean and variance. (8)

12) (a) (i) If X and Y are two random variable having the joint density function

$$f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)}, x,y \geq 0 \\ 0, \text{ otherwise} \end{cases}$$,find the joint density function of $U = \sqrt{X^2+Y^2}$. (8)

(ii) A fair coin is tossed four times. Let X denote the number of heads obtained in the first two tosses and let Y denote the number of heads obtained in the last two tosses. Show that X and Y are independent random variables. (8)

(OR)

(b) Find the correlation coefficient between X and Y for the joint density function

$$f(x,y) = \begin{cases} \frac{6}{7}x, 1 \leq x+y \leq 2, 0 \leq y, x \geq 0 \\ 0, \quad \text{otherwise} \end{cases}$$ (16)

13) (a) (i) The probability that 3 cars will arrive at a parking lot in a 5 minute interval is 0.14. If cars arrive according to a Poisson process, find (1) average arrival of cars, (2)probability that no more than 2 cars arrive in 10 minute interval? (8)

(ii) The random process $X(t) = Y\cos(2\pi t), t \geq 0,$ where Y is a random variable that is uniformly distributed between 0 and 2. Find the expected value and autocorrelation function of $X(t)$. (8)

(OR)

(b) (i) Chennai weather condition can be classified as sunny, cloudy or rainy. A student conducted a detailed study of the weather conditions and came up with the following conclusion: Given that it is sunny on any given day, then on the following day it will be sunny again with probability 0.5, cloudy with probability 0.3 and rainy with probability 0.2. Given that it is cloudy on any given day, then on the following day it will be sunny with probability 0.4, cloudy again with probability 0.3 and rainy with probability 0.3. Given that it is rainy on any given day, then on the following day it will be sunny with probability 0.2, cloudy with probability 0.2 and rainy again with probability 0.3. Determine the limiting-state probabilities of the weather. (8)

(ii) If $\{X(t)\}$ and $\{Y(t)\}$ are two independent Poisson processes, show that the conditional distribution of $\{X(t)\}$ given $\{X(t)+Y(t)\}$ is binomial. (8)

14) (a) (i) The auto correlation function of the Poisson increment process is given by

$R(\tau)=\begin{cases}\lambda^2; & for\ |\tau|>\epsilon\\ \lambda^2+\frac{\lambda}{\epsilon}\left(1-\frac{|\tau|}{\epsilon}\right); & for\ |\tau|\le\epsilon\end{cases}$. Prove that its spectral density is

$$S(\omega)=2\pi\lambda^2\delta(\omega)+\frac{4\lambda\sin^2(\omega\epsilon/2)}{\epsilon^2\omega^2}. \quad (8)$$

(ii) Two jointly wide-sense stationary random processes have sample functions of the form $X(t)=A\cos(\omega_0 t+\Theta)$ and $Y(t)=B\cos(\omega_0 t+\Theta+\varphi)$, where Θ is a uniform random variable in the interval $(0,2\pi)$ and A, B and φ are constants. Find the cross-correlation function $R_{XY}(\tau)$. (8)

(OR)

(b)(i)If $Y(t)=A\cos(\omega_0 t+\Theta)+N(t)$, where A is a constant, Θ is a random variable with a uniform distribution in $(-\pi,\pi)$ and $\{N(t)\}$ is a band limited Gaussian white noise with a power spectral density $S_{NN}(\omega)=\begin{cases}\frac{N_0}{2}, & for\ |\omega-\omega_0|<\omega_B\\ 0, & elsewhere\end{cases}$. Find the power spectral density of $\{Y(t)\}$. Assume that $N(t)$ and Θ are independent. (8)

(ii) Two random processes $X(t)$ and $Y(t)$ are defined by $X(t)=A\cos(\omega t+\Theta)$ and $Y(t)=A\cos(\omega t+\Theta)$, where A and ω are constants and Θ is a uniform random variable on $[0,2\pi]$.Find the cross-correlation and verify that $R_{XY}(-\tau)=R_{YX}(\tau)$. (8)

15) (a)(i) A circuit has an impulse response given by $h(t)=\begin{cases}\frac{1}{T}, & 0\le t\le T\\ 0, & otherwise\end{cases}$. Evaluate $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$. (8)

(ii) A wide sense stationary process $X(t)$ has the autocorrelation function given by $R_{XX}(\tau)=\cos(\omega_0\tau)$. The process is input to a system with the power transfer function $|H(\omega)|^2=\frac{64}{|16+\omega^2|^2}$. Find the power spectral density of the output process. If $Y(t)$ is the output process, find the cross-power spectral density $S_{XY}(\omega)$. (8)

(OR)

(b) A random process $X(t)$ is the input to a linear system whose impulse response is $h(t)=2e^{-t} for\ t\ge 0$. Determine $R_{XY}(\tau)$, $R_{YX}(\tau)$, $S_{XY}(\omega)$ and , $S_{YX}(\omega)$ for the autocorrelation function of the process $R_{XX}(\tau)=e^{-2|\tau|}$. (16)
