Roll No. | | | | | | | | |

B.E./ B.Tech (Part Time) End Semester DEGREE EXAMINATION, APRIL/ MAY 2011

First Semester

Common to All Branches

PTMA 9111 – APPLIED MATHEMATICS

(Regulation 2009)

Time : 3 Hours Answer ALL Questions Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. Two of the eigenvalues of a 3x3 matrix A are 2, 1 and $|A| = 12$. Find the eigenvalues of A^{-1}.
2. If λ is an eigenvalue of A, then prove that $k\lambda$ is an eigenvalue of (kA).
3. Test whether $u = \sin^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ is homogeneous .If so, find its degree.
4. If $u = \frac{y}{x}$ and $v = \frac{x}{y}$, find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.
5. If $u = x^2 - y^2$, then prove that u satisfies Laplace equations.
6. Under the transformation $w = \frac{1}{z}$, find the Image of $|z-i| = 2$.
7. Define an essential singularity of f(z) and give an example.
8. Evaluate $\int_C \frac{dz}{z-a}$ if 'a' is a point inside and outside C.
9. State the sufficient conditions for the existence of Laplace transform.
10. Find inverse Laplace transforms of $\log\left(\frac{s+1}{s}\right)$.

PART-B (5 x 16 = 80 Marks)

11(i) Discuss the maxima and minima of the function $x^4 + y^4 - 2x^2 - 2y^2 + 4xy$. (8)

11(ii) Expand e^{xy} in powers of (x - 1) and (y - 1) up to second degree terms by using Taylor's expansion. (8)

12(a)(i) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and also use it to find A^{-1}. (8)

12(a)(ii) Diagonalize the matrix $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$ by means of an orthogonal transformation. (8)

(OR)

12(b)(i) Find the value of k such that the matrix $\begin{bmatrix} 1 & 2 \\ k & 1 \end{bmatrix}$ has equal eigen values? (4)

12(b)(ii) Reduce the quadratic form $x^2 + 2y^2 + z^2 - 2xy + 2yz$ to canonical form by orthogonal transformation. (12)

13(a)(i) Find the analytic function f(z0 = u + iv, given that $3u + 2v = y^2 - x^2 + 16xy$. (8)

13(a)(ii) Show that the function $v = e^{-x}(x\cos y + y\sin y)$ are harmonic and find a corresponding analytic function $f(z) = u(x,y) + iv(x,y)$. (8)

(OR)

13(b)(i) If $f(z) = u + iv$ is an analytic function of z, prove that $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|f(z)|^2 = 4|f'(z)|^2$. (8)

13(b)(ii) Find the bilinear transformation that maps i, -1, 1 of the z-plane onto 0, 1, ∞ of the w-plane respectively. (8)

■

14(a)(i) Using Cauchy's integral formula, evaluate $\int_C \frac{z}{(z-1)^2(z+2)} dz$, where C is the circle $|z| = \frac{3}{2}$. (6)

14(a) (ii) Evaluate, by using contour integration, the integral $\int_0^\infty \frac{1}{(1+x^2)^3} dx$. (10)

(OR)

14(b)(i) Find the Laurent's series of $f(z) = \frac{z^2-1}{z^2+5z+6}$ valid in the region $2 < |z| < 3$. (6)

14(b)(ii)Evaluate, by using contour integration, the integral $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$. (10)

■

15(a)(i) Find the Laplace transform of $f(t) = \begin{cases} t \text{ for } (0,a) \\ (2a-t) \text{ for } (a,2a) \end{cases}$ with the period $f(t+2a) = f(t)$. (8)

15(a)(ii)Find $L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)$ by using convolution theorem. (8)

(OR)

15(b)(i) Find the Laplace transforms of (1) $te^{-2t}\cos 3t$ and (2) $\frac{e^{-t} - e^{2t}}{t}$. (8)

15(b)(ii) Solve, by using Laplace transform technique,

$y'' - 2y' + y = e^t$ if $y(0) = 2$ & $y'(0) = -1$. (8)

■

****