Roll No. | | | | | | | | |

B.E./ B.Tech (Part Time) End Semester DEGREE EXAMINATION, APRIL/ MAY 2011

Third Semester

Common to CSE & IT

PTMA 9265 – DISCRETE MATHEMATICS

(Regulation 2009)

Time : 3 Hours — Answer ALL Questions — Max. Marks 100

PART-A (10 x 2 = 20 Marks)

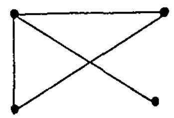
1. Write the statement " The sun is shining and I shall play tensis this afternoon" in symbolic form.
2. If P, Q, R ate three statements with truth values ' true', 'true' and 'false' respectively, find the truth values of $(P \wedge \neg Q) \rightarrow R$ and $(P \wedge \neg Q) \wedge \neg R$.
3. Find the recurrence relation satisfying $y_n = A\,(3)^n + B(-4)^n$.
4. How many four digit numbers (without repetition) can be formed from the six digits 1, 2, 3, 5, 7, 8?
5. Define null graph and complete graph.
6. How many edges are there in a graph with 10 vertices each of degree 5?
7. Test whether the set of positive integers with binary operation addition is a monoid.
8. Find the identity element of the group Z of integers with the binary operation * defined by a * b = a + b +2 for all a, b $\in$ Z.
9. Define a lattice and give an example.
10. Draw the Hasse diagrams of D(45), the lattice of all positive divisors of the integer n.

PART-B (5 x 16 = 80 Marks)

11.(i) Show that a simple graph G with n vertices is connected if G has more than $\frac{1}{2}(n-1)\,(n-2)$. (6)

(ii) Draw a graph with six vertices which is Hamiltonian(Eulerian) but not Eulerian(Hamiltonian). (6)

(iii) Find the adjacency matrix A and the incidence matrix matrix M for the following graph G. (4)



12(a)(i) Verify whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a contradiction or tautology. (8)

(ii) Show that $[\neg P \wedge (\neg Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (8)

(OR)

(b)(i) Find the principal conjunctive normal form and the principal disjunctive normal form of the formula $P \vee [\neg P \rightarrow (Q\ (\neg Q \rightarrow R))\ ]$. (10)

(ii) Use indirect proof, show that $P \rightarrow Q,\ Q \rightarrow R,\ \neg(\ (P \wedge R)\ ,\ P \vee R \Rightarrow R$. (6)

13(a)(i) Show, by the principle of Mathematical induction, that $a^n - b^n$ is divisible by $a - b$ for all $n \in N$.. (6)

(ii) Solve the recurrence relation

$y_n - 4y_{n-1} + 4y_{n-2} = 3n + 2^n$ given $y_0 = 1$ and $y_1 = 1$. (10)

(OR)

(b)(i) Solve, by using generating function, the difference equation

$y_{n+2} - y_{n+1} - 6y_n = 0$ given $y_0 = 2$ and $y_1 = 1$. (8)

(ii) Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2, 3, 5 and 7. (8)

14(a)(i) Show that in a group $\langle G,* \rangle$, if $(a*b)^2 = a^2 + b^2$ for all $a,b \in G$, then G is an abelian group. (6)

(ii) If Z denotes the set of all integers, show that the algebraic system $\langle Z,+,x \rangle$ is aring but not a field, where + and x are the usual addition and multiplication respectively. (10)

(OR)

(b)(i) Show that the Kernel of a homomorphism of a group $\langle G,* \rangle$ into an another group $\langle H,\Delta \rangle$ is a subgroup of G . (8)

(ii) State Lagrange's theorm. Use it to show that if G is a finite group and $a \in G$, then $O(a) \mid O(G)$. (8)

15(a)(i) Prove that a chain is a modular lattice. (6)

(ii) In a Boolean algebra L, show that the De Morgan's laws, given by $(a \wedge b)' = a' \vee b'$ and $(a \vee b)' = a' \wedge b'$ hold for all $a,b \in L$. (10)

(OR)

(b)(i) Let L be a distributive lattice and $a,b,c \in L$. If $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, show that b = c. (6)

(ii) Let L be a complemented distributive lattice. Show that the following are equivalent for each $a,b \in L$, $a \le b \Leftrightarrow a \wedge b' = 0 \Leftrightarrow a' \wedge b = 1 \Leftrightarrow b' \le a'$. (10)

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