Regn. No: | | | | | | | | | | |

B.E/B.Tech(Full Time) DEGREE END SEMESTER EXAMINATIONS, NOV/DEC 2012
COMMON TO ALL BRANCHES
FIRST SEMESTER
MA8151 MATHEMATICS-I
(REGULATION 2012)

Time: 3 Hours

Maximum Marks:100

Answer ALL Questions

Part- A(10x2=20 Marks)

1. If the sum and the product of the eigen values of $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ are 7 and 6. Find a and b?
2. If λ is an eigen value of a square matrix A, then show that λ^2 is an eigen value of A^2.
3. Examine the convergence of the sequence $s_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{n-1}}$.
4. Test the series $\sum_{n=2}^{\infty} \frac{1}{n(\log(n))^2}$ using integral test
5. If $u = xy\log(x)$ and $3x^2 + 3y^2 + 6xy = 1$, find $\frac{du}{dx}$?
6. Suppose that $u = 3x + 2y - z, v = x - 2y + z$ and $w = x(x + 2y - z)$. Are the functions u, v and w functionally related? Why?
7. Evaluate the improper integral $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$.
8. Show that $\frac{d}{dx}\left[erf(ax)\right] = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$
9. Evaluate $\int_0^1 \int_0^x e^{\frac{y}{x}}\ dydx$.
10. Sketch the region of integration in the integral $\int_{\theta=0}^{\pi} \int_{r=2\sin(\theta)}^{4\sin(\theta)} f(r,\theta)\ drd\theta$

Part- B(5x16=80 Marks)

11a. i. Reduce the quadratic form $2x_1x_2 - 2x_1x_3 + 2x_2x_3$ to a canonical form by an orthogonal transformation and discuss its nature (10)

ii. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and hence find the value of $A^{20} - 5A^{11} - 2A^{10} + 3A^2 - 14A - 4I$

(6)

12.a. i. Test the series $\frac{x}{3\sqrt{3}} - \frac{x^2}{3^2\sqrt{5}} + \frac{x^3}{3^3\sqrt{7}} - \cdots$ for absolute convergence and conditional convergence (8)

ii. Test the convergence of the series $\sum_{n=1}^{\infty}(\sqrt{n^4+10} - n^2)$ using comparison test or any one of the test (8)

OR

b. i. For what values of x, the series converges
$\frac{1}{(1-2x)} + \frac{1}{2(1-2x)^2} + \frac{1}{3(1-2x)^3} + \cdots$ (8)

ii. Test the convergence of the series $\frac{1}{4.7.10} + \frac{4}{7.10.13} + \frac{9}{10.13.16} + \cdots$ using comparison test or any one of the test (8)

13.a. i. Expand $e^x \cos(y)$ in powers of x and $(y - \frac{\pi}{4})$ upto second degree. (8)

ii. Find the dimensions of the rectangular box open at the top of maximum capacity whose surface area is 432 sq.cm (8)

OR

b. i. Find the maxima and minima of $x^3 + y^3 + 12xy$ (8)

ii. If $u = sin^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan(u)$ and hence show that
$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin(u)\cos(2u)}{4\cos^3(u)}$ (8)

14. a i. Using differentiation under integral sign, prove that
$\int_0^{\infty} \frac{e^{-\alpha x}\sin(x)}{x} dx = tan^{-1}(\frac{1}{\alpha}), \alpha > 0$. Hence find $\int_0^{\infty} \frac{\sin(x)}{x} dx$? (8)

ii. Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

OR

b. i. Express $\int_0^a x^n(a^m - x^m)^p dx$ in terms of gamma functions and hence evaluate
$\int_0^1 \frac{dx}{\sqrt{(1-x^m)}}$. (8)

ii. Prove that $\beta(m, \frac{1}{2}) = 2^{2m-1}\beta(m, m)$ and hence show that

$\Gamma(m)\Gamma(m + \frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$ (8)

15.a. i. Change the order of integration in $\int_0^1 \int_y^{2-y} xy \, dxdy$ and then evaluate it. (8)

ii. Find the area that lies inside the cardioid $r = a(1 + \cos(\theta))$ and outside the circle $r = a$. (8)

OR

b. i. Evaluate $I = \int\int_R \frac{xy}{\sqrt{(x^2 + y^2)}} dxdy$ by changing into polar coordinates, where R is the region in the first quadrant enclosed by the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4a^2$ (8)

ii. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (8)