Reg. No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

B.E./ B. Tech (Full Time) DEGREE END SEMESTER EXAMINATION, NOV / DEC - 2012

THIRD SEMESTER

MA 9211 - MATHEMATICS III (Regulation - 2008)

COMMON TO ALL BRANCHES

Time: 3 hours **Maximum: 100 Marks**

Answer ALL Questions

Part – A ($10 \times 2 = 20$ marks)

1. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ in $0 < x < 2\pi$, then deduce the value of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$.
2. If $f(x) = \begin{cases} 1-x, & in -\pi < x < 0 \\ 1+x, & in\ 0 < x < \pi \end{cases}$, then find the constant term of the Fourier series of $f(x)$.
3. State and prove modulation theorem on Fourier transforms.
4. Solve for $f(x)$, if $\int_0^{\infty} f(x)\cos\alpha x\, dx = e^{-\alpha}$, where $\alpha > 0$.
5. Obtain the partial differential equation by eliminating the arbitrary function f from $z = e^{-x} f(\frac{y}{x})$.
6. Find the complete solution of $(z - px - qy)(p+q) = 1$.
7. A rod 20cm, long, with insulated lateral surface, has its ends A and B kept at 0°C and 100°C, respectively, find the steady state temperature of the rod, taking the end A at $x = 0$.
8. A uniform rod of length l cm whose surfaces thermally insulated is initially at the temperature $f(x)$. At time $t = 0$, one end $x = 0$ suddenly cooled to zero while the other end $x = l$ thermally insulated. This state is maintained subsequently. Write down the problem mathematically.
9. Find the Z-transform of $\frac{1}{n+1}$.
10. If $Z\{f(n)\} = F(z)$, then prove that $Z\{f(n-k)\} = z^{-k} F(z)$, where k is a positive integer.

Part – B ($5 \times 16 = 80$ marks)

11. i) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & for\ |x| < a \\ 0, & for\ |x| > a > 0 \end{cases}$.

Hence deduce the value of $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$. (8)

ii) By finding the Fourier sine transform of e^{-ax}, for $a > 0$, deduce the values of $\int_0^{\infty}\frac{x\sin sx}{x^2+a^2}dx$ and $\int_0^{\infty}\frac{x^2 dx}{(x^2+a^2)^2}$. (8)

12. (a) i) Find the Fourier series expansion of $f(x) = |\cos x|$ in $(-\pi, \pi)$ of periodicity 2π. (8)

ii) Obtain the half range sine series expansion of $f(x) = x$ in $0 < x < l$.

Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi}{6}$. (8)

(OR)

(b) i) Compute the first two harmonics of the Fourier series of $f(x)$ from the table given (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

ii) Obtain the half range cosine series expansion of $f(x) = x\sin x$ in $0 < x < \pi$.

Hence deduce the value of $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots\ldots to\ \infty$ (8)

13. (a) i) Find the complete solution of $p^2 + x^2y^2q^2 = x^2z^2$ (8)

ii) Find the general solution of $(D^2 - 2DD')z = x^3y + e^{2x-y}$ (8)

(OR)

(b) i) Find the general solution of $px^2 - qy^2 = z(x-y)$ (8)

ii) Find the general solution of $(D^2 + DD' - 6D'^2)z = y\cos x$ (8)

14. (a) A tightly stretched string of length l has its ends fastened at $x = 0, x = l$. The point $x = \frac{l}{3}$ of the string is then taken to a height h and then released from rest in that position. Find the lateral displacement of a point of the string at time t from the instant of release. (16)

(OR)

(b) A rectangular plate with insulated surface is bounded by the lines $x = 0, x = a, y = 0$ and $y = b$. The temperature along the edge $y = b$ kept at $100°C$. While the temperature along the other three edges are at $0°C$. Find the steady- state temperature at any point in the plate. (16)

15. (a) i) Find the inverse Z-transform of $\frac{z^3}{(z-a)^3}$ by using convolution theorem. (8)

ii) Find the inverse Z-transform of $\frac{z(z^2 - z + 2)}{(z+1)(z-2)^2}$, by using residue theorem. (8)

(OR)

(b) i) If $Z\{f(n)\} = \frac{2z^2 + 3z + 12}{(z-1)^4}$, then find $f(2)$ and $f(3)$, by using initial value theorem. (8)

ii) Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = 0, u_1 = 0$, by using Z-transforms. (8)

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