Reg. No. [| | | | | | | | |]

B.E./ B. Tech, DEGREE END SEMESTER EXAMINATION, April / May - 2014

MECHANICAL ENGINEERING

THIRD SEMESTER

MA 8302 – PARTIAL DIFFERENTIAL EQUATIONS

(Regulation – 2012)

Time: 3 hours **Maximum: 100 Marks**

Answer ALL Questions

Part – A ($10 \times 2 = 20$ marks)

1. Obtain the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 - b)$.
2. Find the general solution of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} = z$.
3. Define the complex form Fourier series of $f(x)$, in $(c, c+2l)$.
4. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$, in $0 < x < 2\pi$, then deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$
5. What are the possible solutions of the one dimensional heat flow equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$?
6. What is meant by steady state?
7. Write the second derivative using the second order difference.
8. Write the Crank-Nicholson difference scheme to solve $u_{xx} = au_t$.
9. Write the iterative formula for the solution of system of linear equations in SOR method.
10. Obtain the difference formula for $u_{xx} + u_{yy} = 0$.

Part – B ($5 \times 16 = 80$ marks)

11. i) Solve the following system of linear equations by using Gauss-Elimination method:
$x + 2y + z = 3, 2x + 3y + 3z = 10, 3x - y + 2z = 13$. (8)

ii) Using Crank-Nicholson's scheme, solve $u_{xx} = 16u_t, 0 < x < 1, t > 0$ given $u(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = 100t$. Compute $u(x,t)$ for one step in t direction taking $h = \frac{1}{4}$. (8)

12. (a) i) Find the singular solution of $z = px + qy + p^2q^2$. (8)

ii) Find the general solution of $(D^2 - 2DD' + D'^2)z = \cos(x - 3y) + e^{-2x}$. (8)

(OR)

(b) i) Find the general solution of $z(x - y) = px^2 - qy^2$. (8)

ii) Find the complete solution of $p^2 + q^2 = x^2 + y^2$. (8)

13. (a) i) Find the Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$ of periodicity 2π. Hence deduce the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ......to\ \infty$. (8)

ii) Obtain the half range cosine series expansion of $f(x) = (x-1)^2$ in $0 < x < 1$. (8)

(OR)

(b) i) Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & if\ -\pi < x < 0 \\ x, & if\ \ 0 < x < \pi \end{cases}$. (8)

ii) Obtain the Fourier sine series expansion of $f(x) = x$ in $0 < x < l$. Hence deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (8)

14. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$, find the displacement of the string in the subsequent time. (16)

(OR)

(b) An infinitely long plane uniform plate is bounded by two parallel edges $x = 0$ and $x = l$, and an end at right angles to them. The breadth of this edge $y = 0$ is l and is maintained at a temperature 100° and all the other three edges are at temperature zero. Find the steady state temperature at any interior point of the plate. (16)

15. (a) i) Solve the following system of linear equations by Gauss-Seidel method correct to three decimal places: $28x + 4y - z = 32, x + 3y + 10z = 24, 2x + 17y + 4z = 35$. (8)

ii) Solve $u_{xx} + u_{yy} = 0$ over the square mesh with 1 unit of sub-square of side 4 units, satisfying the following boundary conditions correct to 2 decimal places:

(I) $u(0, y) = 0$, for $0 \le y \le 4$, (II) $u(4, y) = 12 + y$, for $0 \le y \le 4$,

(III) $u(x,0) = 3x$, for $0 \le x \le 4$, (IV) $u(x,4) = x^2$, for $0 \le x \le 4$. (8)

(OR)

(b) i) Using Gauss-Jacobi method, solve the system of equations three decimal places:
$27x + 6y - z = 85, 6x + 15y + 2z = 72, x + y + 54z = 110$. (8)

ii) Solve the Poisson's equation $\nabla^2 u = 8x^2 y^2$ for the square mesh given $u = 0$ on the 4 boundaries dividing the square into 16 sub-squares of length 1 unit. (corrected to three decimal places) (8)

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