Reg. No. ☐☐☐☐☐☐☐☐☐☐☐☐☐

B.E. / B.Tech. END SEMESTER EXAMINATIONS, DECEMBER 2011

I SEMESTER

MA 9111 MATHEMATICS-I
(Common to all Branches)

Time: 3 Hours **Answer All the Questions** **Max. Marks: 100**

Part A **($10 \times 2 = 20$)**

1. Using the properties of eigenvalues, find the eigenvalues of $\text{Adj}(A)$ if the matrix $(A)_{3x3}$ has two eigenvalues 1, 1 and $\det(A) = 4$.

2. Give the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$.

3. State the D'Alembert's ratio test for convergence of a positive term series.

4. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

5. Using Euler's theorem, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$.

6. State the chain rule for Jacobians involving functions of two variables.

7. Find the Cauchy's principal value of the improper integral $\int_{-1}^{1} \frac{dx}{x^{2/3}}$.

8. Compute $\Gamma\left(\frac{9}{2}\right)$ given $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

9. Sketch the region of integration of the integral $\int_{0}^{a} \int_{a-x}^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$.

10. Change the following double integral into polar form $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$.

Part B **(5 × 16 = 80)**

11 (i) Using Cayley Hamilton theorem, find the inverse of the matrix A, where

$$A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}.$$

(8 marks)

(ii) For the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$, verify the property that the eigenvectors corresponding to distinct eigenvalues of a real matrix are linearly independent.

(8 marks)

12a. (i) Find the interval of convergence of the power series

$$\frac{1}{1-x} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + \cdots \infty.$$

(10 marks)

(ii) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$.

(6 marks)

(OR)

12b.(i) Test for conditional convergence of the following series:

$$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4)+\cdots\infty.$$ (10 marks)

(ii) Test the convergence of the following series using D'Alembert's ratio test:

$$\sum_{n=1}^{\infty}\frac{n!}{n^n}.$$ (6 marks)

13a. (i) If $u=\dfrac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$. Find also a relation between u and v. (8 marks)

(ii) Expand $f(x,y)=e^x\cos y$ in powers of x and y as far as the terms of third degree. (8 marks)

(OR)

13b. (i) If $v=\left(x^2+y^2+z^2\right)^{-1/2}$, then find $\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}+\dfrac{\partial^2 v}{\partial z^2}$. (8 marks)

(ii) Given $x+y+z=a$, find the maximum value of $x^m y^n z^p$. (8 marks)

14a. (i) Express the integral $\displaystyle\int_0^{\pi/2}\frac{\sqrt[3]{\sin^8 x}}{\sqrt{\cos x}}\,dx$ in terms of gamma functions.

(10 marks)

(ii) Test for convergence of the improper integral $\displaystyle\int_0^1\frac{x\,dx}{\sqrt{1-x^2}}$. (6 marks)

(OR)

14b. (i) Evaluate $\int_0^{\infty} x^n e^{-\alpha x} dx$ using Leibnitz's rule. (10 marks)

(ii) Show that $\frac{d}{dx}\left[erfc(ax)\right] = -\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$. (6 marks)

15a. (i) Find by double integration, the area inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$. (8 marks)

(ii) Find the volume bounded by the *xy*-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$. (8 marks)

(OR)

15b. (i) Evaluate $\int_0^a \int_0^x \int_0^{x-y} e^{(x+y+z)} dx dy dz$. (8 marks)

(ii) Using the transformation $x + y = u,\ y = uv$, evaluate $\int_0^1 \int_0^{1-x} e^{\left(y/(x+y)\right)} dy dx$.

(8 marks)