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ANNA UNIVERSITY, CHENNAI - 600 025
 B.E / B.TECH DEGREE END SEMESTER EXAMINATIONS – NOV / DEC 2011
 FIRST SEMESTER, REGULATION - 2004
 MA 171 - MATHEMATICS – I
 (COMMON TO ALL BRANCHES)

Time : 3 Hours

Answer ALL Questions

Max. Marks : 100

Part A

(10 x 2 = 20)

1. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the

third eigenvalue.

2. Find the nature of the quadratic form $2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_3x_1$.
3. Find the equation of the sphere whose centre is (1, -2, 3) and which passes through the point (3, 1, -3).
4. Find the equation of the sphere with centre at (1, 2, 3) which touches the z-axis.
5. Explain the monotonic sequence and what is the condition for convergence.
6. Examine the convergence of the series

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \dots \infty$$

7. Find $\frac{dy}{dx}$ using partial differentiation, if $x^3 + 3x^2y + 6xy^2 + y^3 = 1$.
8. If $u = 2xy$, $v = x^2 - y^2$, $x = r \cos \theta$, $y = r \sin \theta$ then compute $\frac{\partial(u, v)}{\partial(r, \theta)}$.
9. Find the particular integral of $(D^2 + 4)y = x^4$.
10. Convert the equation $xy'' - 3y' + x^{-1}y = x^2$ as a linear equation with constant coefficients.

Part-B

(5x16 = 80)

11. i). Test the convergence of the series using D'Alembert's ratio test.

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \dots + \infty. \quad (8)$$

ii). State the values of x for which the following series converge.

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots + \infty. \quad (8)$$

12. a.i). Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (8)

ii). Using Cayley - Hamilton theorem , find the inverse of the given matrix

$$A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} \quad (8)$$

(OR)

b). Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ to a canonical form by orthogonal reduction. Find also a set of non-zero values of x_1, x_2, x_3 which will make the Q.F zero. (16)

13. a.i). Find the equation of the sphere passing through the four points (1,2,3) ,

$$(0,-2,4), (4,-4,2) \text{ and } (3,1,4). \quad (8)$$

ii). Find the equation of the sphere having the circle

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, \quad x + y + z = 3. \text{ as a great circle.} \quad (8)$$