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# Anna University

**B.E /B.Tech (Full Time) Degree Examination, November 2011**

**Third Semester**

**MA 9211 Mathematics-III** (Common to all branches )

**(Regulations 2008)**

Time: Three Hours Maximum: 100 marks

Answer All Questions

PART-A(10 X 2=20 marks)

1. What are the conditions for the Fourier series expansion of a given function f(x) in the given interval $(0, 2l)$.

2. If the half range cosine series of the function f(x) = x of period $2l$ in $(0, l)$ is given by $x = \frac{l}{2} - \frac{4l}{\pi^2}\left(\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \frac{1}{5^2}\cos\frac{5\pi x}{l} + ........\right)$, then find the value of $\sum_{n=!}^{\infty}\frac{1}{(2n-1)^4}$.

3. Prove that F[ f(x) cosax]= $\frac{1}{2}$ [F(s+a)+F(s-a)] where F(s) = Fourier transform of f(x).

4. Evaluate $\int_0^{\infty}\frac{s^2 ds}{(s^2+a^2)(s^2+b^2)}$ using suitable Fourier transform technique.

5. Form the partial differential equation by eliminating the arbitrary functions f and g from $z = x^2 f(y) + y^2 g(x)$.

6. Solve $(D + D' - 1)(D + 2D' - 3)z = 0$.

7. Write down all the possible trial solutions of the two dimensional heat equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

8. Solve the equation $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given that $u(0,y) = 8e^{-3y}$ by the method of separation of variables.

9. Find the Z-transform of $\frac{1}{n+1}$.

10. State initial value theorem and final value theorem on Z-transform.

PART-B (5 x 16 =80 marks)

11.( a.) (i)Solve the difference equation using Z-transform technique
$y_{n+2} + 4\,y_{n+1} + 3\,y_n = 2^n$ with $y_0 = 0, y_1 = 1$ . (8)

(ii) Using inversion integral, find the inverse Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$. (4)

(iii) Using convolution theorem, find $Z^{-1}\left(\frac{z^2}{(z^2-5z+6)}\right)$ . (4)

12.( a.) (i) Obtain a Fourier series for a function f(x)= $x^2$ $-\pi \le x \le \pi$ .

Deduce the sum of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$. (8)

(ii) The following table gives the variations of periodic current over a period

| t(secs): | 0 | $\frac{T}{6}$ | $\frac{T}{3}$ | $\frac{T}{2}$ | $\frac{2T}{3}$ | $\frac{5T}{6}$ | T |
|---|---|---|---|---|---|---|---|
| A(amp): | 1.98 | 1.30 | 1.05 | 1.30 | $-0.88$ | $-0.25$ | 1.98 |

Express A in a Fourier series as far as the second harmonic using the above data and also obtain the amplitude of the first harmonic . (8)

**OR**

( b) (i) Find the Fourier series expansion for f(x) , if f(x)= $\begin{cases} -\pi, -\pi \prec x \prec 0 \\ x, 0 \prec x \prec \pi \end{cases}$

and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+........=\frac{\pi^2}{8}$ . (10)

(ii) Find the complex form of the Fourier series of the periodic function
f(x) = cosax $-\pi <. x < \pi$ . (6)

13.( a) (i) Find the Fourier transform of f(x)= $\begin{cases} 1-|x|, |x|<1 \\ 0, |x|>1 \end{cases}$

and hence solve $\int_0^{\infty}\frac{\sin^4 t}{t^4}dt$ . (8)

(ii) Find the Fourier cosine transform of $e^{-x^2}$ and hence evaluate Fourier sine transform of $x\,e^{-x^2}$ . (8)

**OR**

(b) (i) Solve the integral equation

$$\int_0^\infty f(x)\cos sx\,dx = \begin{cases} 1-s, 0 \le s \le 1 \\ 0, s > 1 \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. (8)

(ii) Verify Convolution theorem for Fourier Transform if $f(x) = g(x) = e^{-x^2}$. (8)

14. (a) (i) Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. (8)

(ii) Solve $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$. (8)

**OR**

(b) (i) Solve $z^2(p^2 + q^2) = (x^2 + y^2)$. (6)

(ii) Find the integral surface of the equation satisfying the equation $x(y^2+z)p - y(x^2+z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$. (10)

15. (a) A tightly stretched string of length $l$ has its ends fastened at $x = 0$, $x = l$. The mid-point of the string is taken to a height h and then released from rest in that position. Find the displacement of the string at any time t. (16)

OR

(b) A bar of $l$ cm long, with insulated sides has its ends A and B maintained at temperatures $50^0$ C and $100^0$ C respectively, until steady-state conditions prevail. The temperature at A is suddenly raised to 90 $^0$C and at the same time that at B is lowered to $60^0$ C. Find the temperature distribution in the bar at time t.

(16)