REG. NO.

B.E. / B.TECH (FULL TIME) DEGREE END SEMESTER EXAMINATION,

NOVEMBER 2011

CIVIL ENGINEERING

THIRD SEMESTER

MA 271 MATHEMATICS III

(COMMON TO ALL BRANCHES)

(REGULATION 2004)

TIME: THREE HOURS **MAX.:100 MARKS**

Answer ALL Questions

PART- A (10 x 2 = 20 Marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 - b)$.
2. Find the complete solution of $z = pq$.
3. The half-range sine series of $x(\pi - x)$ in $[0, 2\pi]$ is $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{2}\right)$. Find $\sum_{n=1}^{\infty} b_n^2$.
4. Write the complex form of Fourier series for f(x) defined in $c < x < c + 2\ell$.
5. Write down the initial conditions when a taut string of length 2ℓ is fastened on both ends. The midpoint of the string is taken to a height b and released from the rest in that position.
6. What are the possible solutions for $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by method of separation of variables?
7. If $F[f(x)] = F(S)$, then prove that $F[f(x)\cos ax] = \frac{1}{2}[F(S-a) + F(S+a)]$.
8. Find the Fourier cosine transform of $f(x) = 5e^{-2x}$.
9. Find $Z\left(\frac{1}{n!}\right)$ and using this find $Z\left(\frac{1}{(n+1)!}\right)$.
10. If $Z(u_n) = U(z)$, then prove that $Z(nu_n) = -z\frac{dU(z)}{dz}$.

PART- B (5 x 16 = 80 Marks)

11.(i) Find the general solution of $(2z - y)p + (x + z)q + 2x + y = 0$. (8)

11.(ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = e^{2x+y} + \sin(x + 2y)$. (8)

12(a)(i) Find the half range cosine series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x & 1 < x < 2 \end{cases}$. (8)

12(a)(ii) Obtain the constant term and the co-efficient of the first sine and cosine terms in the Fourier expansion of f(x) for the data given below:

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

(8)

(OR)

12(b) Find the Fourier series for $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$ and hence deduce the value of $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7}$ .... (16)

13(a) A rod of length 20 cm has its ends A and B kept at 30°C and 90°C respectively, until steady state conditions prevail. If the temperature at each end is then suddenly reduced to 0°C and maintained so, find the temperature u(x,t) at a distance x from A at time t. (16)

(OR)

13(b) A rectangular plate is bounded by the lines x = 0, x = a, y = 0 and y = b and the edge temperatures are u(0,y) = 0,u(x,b) = 0,u(a,y) = 0 and u(x,0) = $5\sin\left(\frac{5\pi x}{a}\right)+3\sin\left(\frac{3\pi x}{a}\right)$. Find the steady state temperature distribution u(x,y) at any point of the plate. (16)

14(a) Find the Fourier transform of $f(x)=\begin{cases}1 & \text{if } |x|<a \\ 0 & \text{if } |x|>a>0\end{cases}$. Hence evaluate the integrals (i) $\int_0^\infty \frac{\sin t}{t}dt$ and (ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$. (16)

(OR)

14(b) Find the Fourier sine and cosine transforms of e^{-x} and hence find the Fourier sine transform of $\frac{x}{x^2+1}$ and Fourier cosine transform of $\frac{1}{x^2+1}$. (16)

15(a)(i) From the relation $y_n = a2^n + b(-2)^n$, derive a difference equation by eliminating the constants a and b. (8)

15(a)(ii) Using method of partial fraction find $Z^{-1}\left[\frac{z^3}{(z-1)^2(z-2)}\right]$. (8)

(OR)

15(b)(i) Use convolution theorem to evaluate the inverse Z-transform of $\frac{z^2}{(z-1/2)(z-1/4)}$. (6)

15(b)(ii) Solve, by Z-transform method, the equation $y_{n+2}+4y_{n+1}+3y_n=3^n$, given $y_0=0, y_1=1$. (10)

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