Roll No. | | | | | | | | | |

**B.E./B.Tech (Full Time) END SEMESTER EXAMINATIONS, NOV / DEC 2013**

**SECOND SEMESTER – (REGULATIONS: 2004/ 2008)**

**COMMON TO ALL BRANCHES**

**GE 181 / GE 9151 ENGINEERING MECHANICS**

**Time: 3hrs** **Max. Marks: 100**

**Answer All Questions**

**PART-A** (10 x 2 = 20 Marks)

1 What are the equations of equilibrium for a rigid body acted upon by a system of forces?

2 What is meant by angle of friction? What is its relation to friction coefficient and angle of repose?

3 Determine whether the systems of forces shown in Fig.3 are in equilibrium?

4 State and explain Varignon's theorem?

5 Locate the centroid of the lamina shown in the Fig.5.

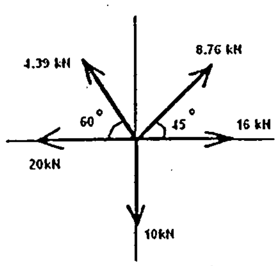



**Fig.3**

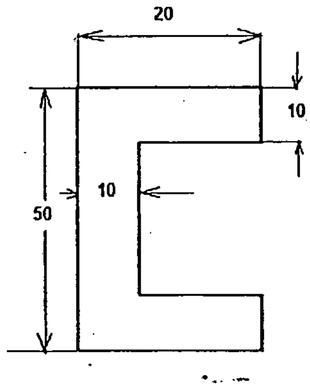



**Fig.5**

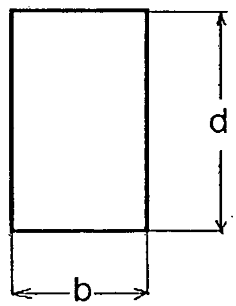



**Fig.6**

6 State parallel axes theorem and use it to determine the moment of inertia of the rectangle shown in Fig.6 about its bottom edge.

7 A man weighing 750N is travelling in a lift with an acceleration of $2m/s^2$. What is the force exerted by the man on the floor when moving (i) upwards (ii) downwards.

8 An aircraft of mass 10000kg is flying at a velocity of 300 km/ph at a height of 3000m above ground level. Calculate the total energy possessed by the aircraft.

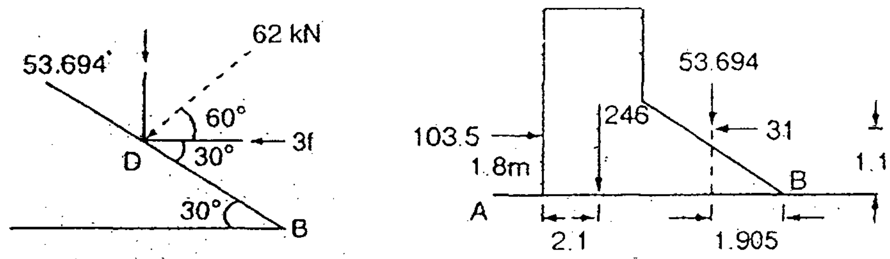
9 Explain clearly what is coefficient of restitution.

10 With a neat sketch explain what is centre of percussion.

**PART-B** (5 x 16 = 80 Marks)

11. i) Two cylinders of diameters 60mm and 30mm and weighing 160N and 40N respectively are placed as shown in Fig.11.a. Assuming all the contact surfaces to be smooth, find the reactions at A,B and C.

ii) The three cables are secured to a ring at B and the turn buckle at C as shown in Fig 11.b. is tightened until it supports a tension of 1.6 kN. Calculate the moment M, produced by the tension in cable AB about the base of the mast at D.




(b) components of 62 kN at 'D' (c)

*Figure 4.22*

The components of forces are shown in Fig. 4.22(b).

$$\overset{+}{\rightarrow} \Sigma F_X = 103.5 - 31 = 72.5$$

$$(+\uparrow) \Sigma F_Y = -246 - 53.694 = -299.694$$

$\Sigma F_X$ is positive and $\Sigma F_Y$ is negative; hence 'R' is in fourth quadrant.

$$R = \sqrt{(72.5)^2 + (299.694)^2} = 308.339$$

$$\tan\alpha = \frac{299.694}{72.5};\ \alpha = 76.4°.$$

$$(+) \Sigma M_A = -103.5(1.8) - 246(2.1) - 53.694(3.795) + 31(1.1)$$
$$= -872.569 \text{ kNm}$$

$$d = \frac{M_A}{R} = \frac{872.569}{308.339} = 2.83 \text{ m}$$

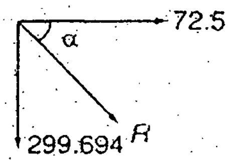


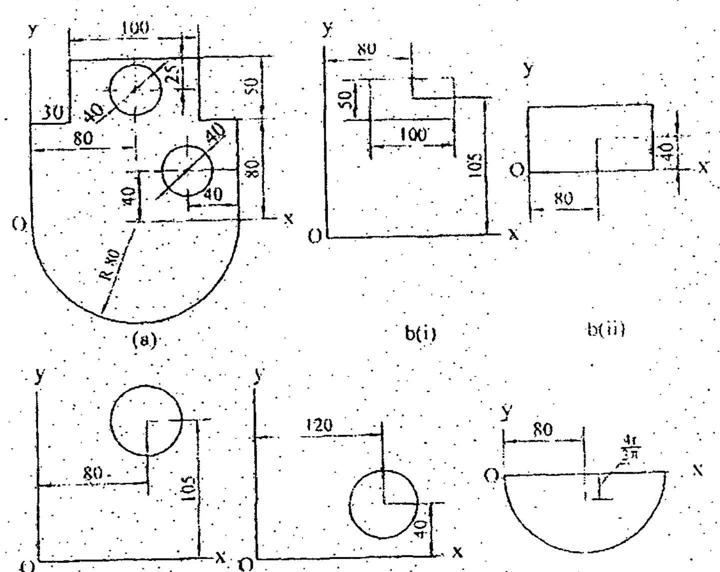

*Figure 4.22(d)*

$\overline{X}$ from A;

$$\overline{X}_A = \frac{872.569}{299.694} = 2.912 \text{ m}$$

$A_1B_1$ is middle third region *i.e.*, from A 1.9 m to 3.8 m. The resultant passes through 2.912 m from A which lies within middle third region. Hence the design is safe.

13.a




(a) b(i) b(ii)

b(iii) b(iv) b(v)

| Component | Area (mm²) | Distance of its C.G. from $Ox$ (mm) | Distance of its C.G. from $Oy$ (mm) | Area × distance from $Ox$ (mm³) | Area × distance from $Oy$ (mm³) |
|---|---|---|---|---|---|
| Small rectanlge | $100 \times 50 = 5 \times 10^3$ | 105 | 80 | $525 \times 10^3$ | $+400 \times 10^3$ |
| Big rectangle | $160 \times 80 = 12.8 \times 10^3$ | 40 | 80 | $+512 \times 10^3$ | $+1024 \times 10^3$ |
| Circle | $-\pi \times 20^2 = -1.257 \times 10^3$ | 105 | 80 | $-132 \times 10^3$ | $-100.56 \times 10^3$ |

| Component | Area | Distance of its C.G. from $Ox$ | Distance of its C.G. from $Oy$ | Area × distance from $Ox$ | Area × distance from $Oy$ |
|---|---|---|---|---|---|
| Circle | $-\pi \times 20^2 = -1.257 \times 10^3$ | 40 | 120 | $-50.27 \times 10^3$ | $-150.84 \times 10^3$ |
| Semicircle | $\frac{1}{2}\pi \times 80^2 = 10.053 \times 10^3$ | $\frac{-4r}{3\pi} = -33.95$ | 80 | $-341.33 \times 10^3$ | $+804.24 \times 10^3$ |
| | $\Sigma$ Area $= 25.34 \times 10^3$ | | | $\Sigma$ Moment of areas about $Ox = 513.4 \times 10^3$ | $\Sigma$ Moment of areas about $Oy = 1976.84 \times 10^3$ |

$$\bar{x} = \frac{\Sigma \text{ First moment of area about } Oy}{\Sigma \text{ Area}} = \frac{1976.84 \times 10^3}{25.34 \times 10^3} \text{ mm} = 78.01 \text{ mm}$$

$$\bar{y} = \frac{\Sigma \text{ First moment of area about } Ox}{\Sigma \text{ Area}} = \frac{513.4 \times 10^3}{25.34 \times 10^3} \text{ mm} = 20.26 \text{ mm}$$

**$O_y$ = 78.01 mm , $O_x$ = 20.26 mm**

13.b.i

**To find the position of *YY***

Let $x$ be the perpendicular distance of the centroid from $PQ$

Then, $x = \frac{16 \times 4 \times 8 + 12 \times 4 \times 2 + 16 \times 4 \times 8}{16 \times 4 + 4 \times 12 + 16 \times 4}$

$= \frac{512 + 96 + 512}{64 + 48 + 64} = \frac{1\,120}{176} = 6.36$ cm

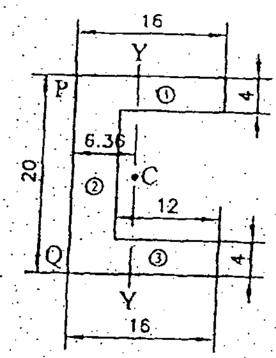


Fig. 8

*MI* of the channel section about $Y - Y$ =

Sum of the MI of the rectangles 1, 2 and 3 about $Y - Y$

$I_{YY}$ of rectangle 1 $= \frac{1}{12} \times 4 \times 16^3 + 4 \times 16 \times 1.64^2 = 1\,365.33 + 172.13 = 1537.46$ cm$^4$

$I_{YY}$ of rectangle 2 $= \frac{1}{12} \times 12 \times 4^3 + 12 \times 4 \times 4.36^2 = 64 + 912.46 = 976.46$ cm$^4$

$I_{YY}$ of rectangle 3 $= 1\,537.46$ cm$^4$

$I_{YY}$ of the given section $= 2 \times 1\,537.46 + 976.46 = 4\,051.38$ cm$^4$

**Answer :** $I_{YY}$ of the given section $= 4\,051.38$ cm$^4$

14.a.i

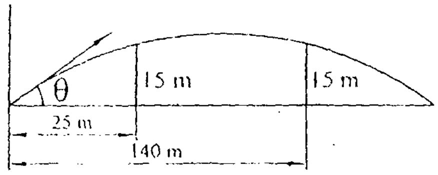



Fig. 3

*Solution*

The equation of trajectory is used here.

$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$15 = 25 \tan\theta - \frac{9.81 \times 25^2}{2u^2\cos^2\theta} \quad \text{.... (1)}$$

$$15 = 140 \tan\theta - \frac{9.81 \times 140^2}{2u^2\cos^2\theta} \quad \text{.... (2)}$$

(1) × 140 gives $2\,100 = 3\,500 \tan\theta - \frac{9.81 \times 25^2}{2u^2\cos^2\theta} \times 140$ .... (3)

(2) × 25 gives $375 = 3\,500 \tan\theta - \frac{9.81 \times 140^2}{2u^2\cos^2\theta} \times 25$ .... (4)

(3) − (4) gives $\frac{(49 \times 10^4 - 8.75 \times 10^4)\, 9.81}{2u^2\cos^2\theta} = 1.725$ or $2u^2\cos^2\theta = 2\,289$

Substituting $2u^2\cos^2\theta = 2\,289$ in eqn. (1), we get $\theta = 35.26°$

Substituting for $\theta$ in eqn. (5), we get $u = 41.43$ m/s

**Answers :** Angle of projection = 35.26°. Velocity of projection = 41.43 m/s.

14.a.ii Fig. 5 shows the forces acting on the car. Weight and normal reaction are equal and opposite in direction. Force on the tyres should be sum of the forces along the radius.

$m = 1200$ kg, $v = 125$ m/s, $r = 400$ m

$$= ma_n = \frac{mv^2}{r} = \frac{1\,200 \times (12.5)^2}{400} = 468.75 \text{ N.}$$

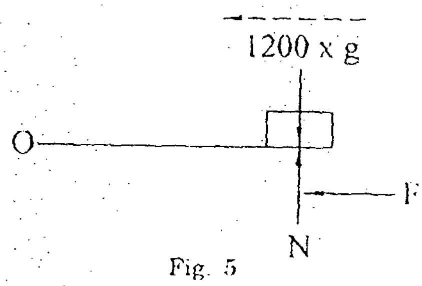



Fig. 5

**Answer :** Force on the tyres = 468.75 N.

13.b.ii

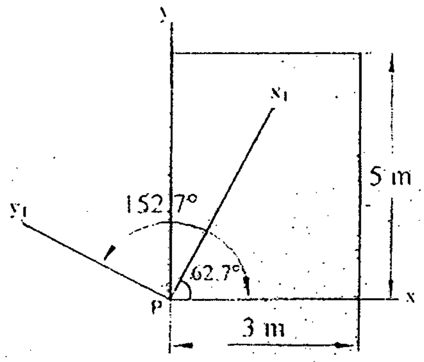



Fig. 6

*Solution*

$I_{xx}$ of the rectangle $= \frac{1}{3} \times b h^3 = \frac{1}{3} \times 3 \times 5^3 = 125 \text{ m}^4$

$I_{yy}$ of the rectangle $= \frac{1}{3} \times h b^3 = \frac{1}{3} \times 5 \times 3^3 = 45 \text{ m}^4$

$I_{xy}$ of the rectangle with respect to $x$ and $y$ axes $= \frac{b^2 h^2}{4} = \frac{3^2 \times 5^2}{4} = 56.25 \text{m}^4$

**To find the location of the principal axes**

Let $\theta_1$ and $\theta_2$ be the inclination of the principal axes with the $x$ axis.

$$\text{Then, } \tan 2\theta_1 = -2 \frac{I_{xy}}{I_{xx} - I_{yy}} = \frac{-2 \times 56.25}{125 - 45} = \frac{-112.5}{80} = -1.406$$

$$2\theta_1 = -54.6^\circ \text{ or } 305.4^\circ \text{ or } \theta_1 = 152.7^\circ ; \theta_2 = 152.7^\circ - 90^\circ = 62.7^\circ$$

## The Principle Moment of Inertia

$$I_{\max} = \frac{I_{xx} + I_{yy}}{2} + \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}^2}$$

$I_{max} = 154 \text{ m}^4$ , $I_{min} = 16 \text{ m}^4$

15.b i) Two blocks of weight 1 kN and 2kN are kept on a horizontal plane as shown in Fig.15.b.i. The coefficient of friction between the blocks is 0.25 and that between 2kN block and the plane is 0.3. Find the minimum force A required to just move the 2kN block when ( a) the cable is tied to the 1 kN block tightly (b) the cable is removed.

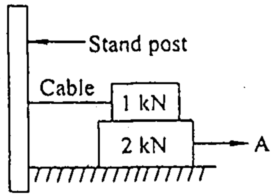



Fig.15.b.i

ii) A small pulley of radius 100mm is connected to the shaft of an electric motor. A belt connects this pulley with a bigger pulley of radius 300mm. Contact angle between the bigger pulley and the belt is 230°. The maximum permissible tension in the belt is 2000N. The coefficient of friction $\mu_s$ between the belt and both pulleys is 0.25. Find the torque exerted by the belt on the bigger pulley. Also check that $\mu_s$ utilised in the case of bigger pulley at the time of slipping in the smaller pulley is less than 0.25.

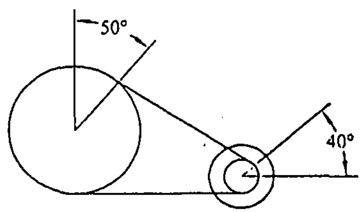



Fig.15.b.ii

5

14.b.i

$$\therefore \mu = (F/N) = \left(\frac{36.788}{245.25}\right) = 0.15$$

When 1.25 kg is added to 3.75 kg mass, it moves down.

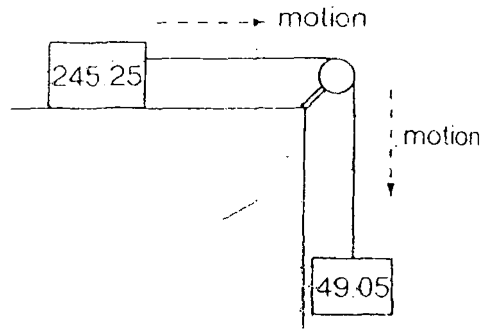



*Figure 11.22(c)*

$$\therefore \quad W = 5 \times 9.81 = 49.05\,\text{N}$$

$$\Sigma F_x = 0 : -25a - 0.15N_A + T = 0$$

$$\Sigma F_y = 0 : N_A - 245.25 = 0;\ N_A = 245.25$$

$$\therefore T - 25a = 0.15(245.25)$$

$$T - 25a = 36.788$$

$$\Sigma F_y = 0 : T + 5a - 49.05 = 0$$

$$T + 5a = 49.05$$

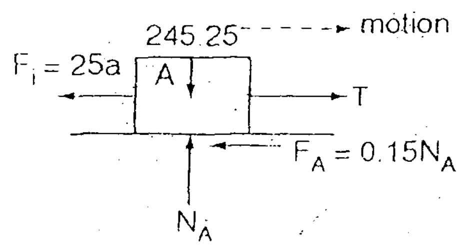



*Figure 11.22(d)*

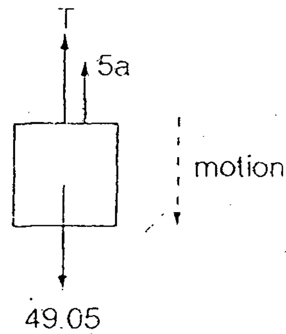



*Figure 11.22(e)*

Solving the above two equations

$$30a = 12.262;\ a = 0.4087\,\text{m/s}^2$$

$$\therefore \quad T = 25(0.4087) + 36.788 = 47.006\,\text{N}$$

Therefore acceleration of masses $a = 0.409\,\text{m/s}^2$ and Tension in string, $T = 47.006$ N

14.b.ii

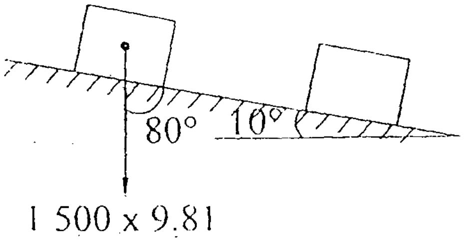



Fig 5

***Solution***

Work done by the force = change in K.E. of the car.

$$\text{Initial kinetic energy} = \frac{1}{2} \times 1\,500 \times v^2 = \frac{1}{2} \times 1\,500 \times (16.67)^2 = 208.416 \times 10^3 \text{ J}$$

$$\text{Final kinetic energy} = \frac{1}{2} \times 1\,500 \times 0^2 = 0$$

Work done by the force on the car = work done by the applied force + work done by the self-weight of the car.

$$= -5\,000 \times x + 1\,500 \times 9.81 \times \cos 80° \times x$$

Here, (–) sign is attached to the force of 5 kN since it is acting in the direction opposite to that of motion.

$$\text{Work done} = -5\,000\,x + 2\,555.23\,x = \;\; 2\,444.77x$$

Using the principle of work and energy, we get $2444.77\,x = 0 - 208.416 \times 10^3$

$X$ = 85.25 m.

15.a.i

*Solution*

Here, the principle of conservation of momentum is applied since there are no external forces acting on the system in the plane of motion.

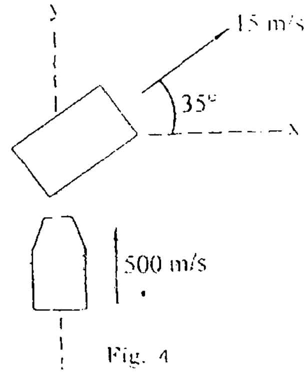


Fig. 4

Initial momentum of the block in $x$ direction = $5 \times 15 \times \cos 35$

Initial momentum of the bullet in $x$ direction = 0

Final momentum of the combined unit in $x$ direction

= $(5 + 0.04)\, v_x$ where $v_x$ is the $x$ component of the velocity of the combined unit after impact

Initial momentum of the system in $x$ direction = final momentum of the system in $x$ direction

or $5 \times 15 \times \cos 35° + 0 = (5.04)\, v_x$

or $v_x = 12.19$ m/s

Initial momentum of the block in $y$ direction = $5 \times 15 \times \sin 35°$

Initial momentum of the bullet in $y$ direction = $0.04 \times 500$

Final momentum of the combined unit in $y$ direction = $5.04 \times v_y$ where $v_y$ is the $y$ component of the velocity of the combined unit after impact.

Initial momentum of the system in $y$ direction = final momentum of the system in $y$ direction.

or $5 \times 15 \times \sin 35° + 0.04 \times 500 = 5.04 \times v_y$ or $v_y = 12.5$ m/s

Velocity of the combined unit after impact = $\sqrt{(v_x)^2 + v_y^2} = \sqrt{(12.19)^2 + (12.5)^2}$

$= 17.46$ m/s

Inclination of the final velocity with $x$ axis = $\tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\frac{12.5}{12.19} = 45.7°$

**Answer :** The velocity of the combined unit after impact = 17.46 m/s making an angle of 45.7° counter-clockwise with $x$ axis

15.a.ii

To find the velocity of the ball before impact

$$v = \sqrt{2gh} = 14 m/s$$

To find the velocity of the ball after impact

$$v = -\sqrt{2gh} = -11.29 m/s$$

Coefficient of restitution between the ball and floor

$$= \frac{\text{velocity of the floor after impact} - \text{velocity of the ball after impact}}{\text{velocity of the ball before impact} - \text{velocity of the floor before impact}} = 0.806$$

Successive rebound of a body

$$e = \frac{\text{velocity of body after impact}}{\text{velocity of body before impact}} = \frac{v_2}{v_1}$$

Expressing e in terms of height, $e = \frac{\sqrt{2gH_1}}{\sqrt{2gH}}$

Squaring both sides, $e^2 = \frac{2gH_1}{2gH} = \frac{H_1}{H}$

This equation can be written for any number of rebounds.

For the first rebound, $e^2 = \frac{H_1}{H}$, For second rebound, $e^2 = \frac{H_2}{H_1}$ where $H_2$ is the height of rebound after the second impact and $H_1$ is the height of rebound after first impact.

15.b.i

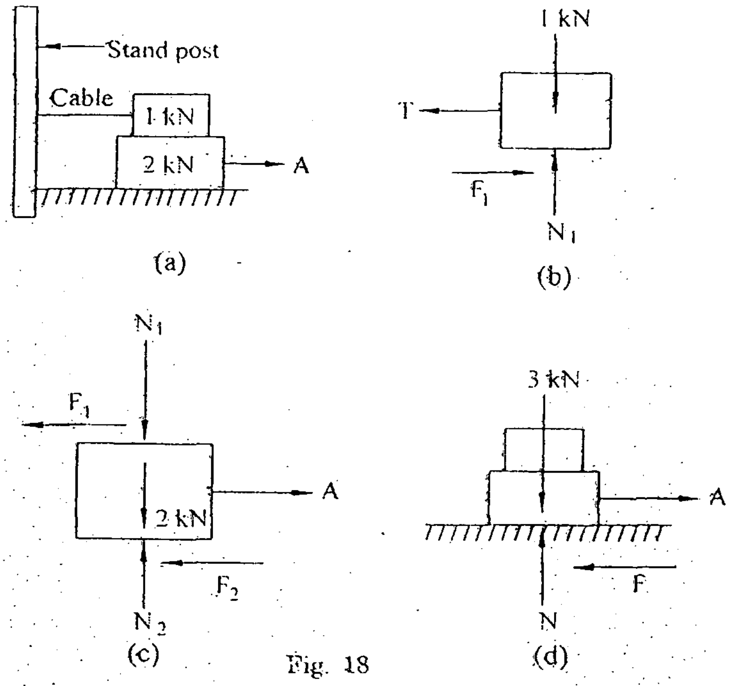



Fig. 18

*Solution*

**Equilibrium of 1 kN block**

Let $T$ be the tension in the cable. The free-body diagram of 1 kN block is shown in Fig. 18 (b). When the 2 kN block is just moved to the right by a force $A$, 1 kN block moves to the left since it is tied to the cable. Hence, the friction between the blocks acts to the right thus opposing the leftward movement of 1 kN block.

$$\Sigma F_x = F_1 - T = 0 \text{ or } F_1 = T.\ \Sigma F_y = N_1 - 1\text{ kN} = 0 \text{ or } N_1 = 1\text{ kN}$$

$$F_1 = \mu N_1 = 0.25 \times 1\text{ kN} = 0.25\text{ kN}.\ \text{Tension in the cable } T = F_1 = 0.25\text{ kN}$$

**Equilibrium of 2 kN block**

The free-body diagram of 2 kN block is shown in Fig. 18 (c). The friction between the two blocks acts opposite to the direction of motion. Friction between the 2 kN block and the plane also opposes motion.

$$\Sigma F_x = A - F_1 - F_2 = 0 \text{ or } A = F_1 + F_2$$

$$\Sigma F_y = N_2 - N_1 - 2\text{ kN} = 0 \text{ or } N_2 = N_1 + 2\text{ kN} = 1\text{ kN} + 2\text{ kN} = 3\text{ kN}\ (\because N_1 = 1\text{ kN})$$

But $F_2 = 0.3 \times N_2 = 0.3 \times 3\text{ kN} = 0.9\text{ kN}.\ A = F_1 + F_2 = 0.25\text{ kN} + 0.90\text{ kN} = 1.15\text{ kN}$

**Value of $A$ in the absence of the cable** (Fig.18 d)

$$N = 3\text{ kN and } A = F.\ \text{But } F = \mu \times 3\text{ kN} = 0.3 \times 3\text{ kN}.\ \therefore A = F = 0.9\text{ kN}$$

**Answers**

(i) The minimum force $A$ required to just move the 2 kN block with the cable is 1.15 kN.

(ii) The minimum force $A$ required to just move the 2 kN block without the cable is 0.9 kN.

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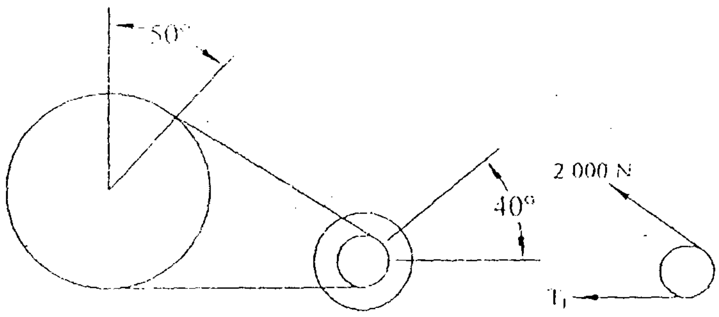
15.b.ii




***Solution*** (Fig. 28)

Resistance to slipping depends on the angle of contact. Here, $\mu_s$ is the same for both the small and big pulleys. The contact angle between the belt and the bigger pulley is 230°. Contact angle between the belt and the smaller pulley is 90° + 40° = 130°. Hence, slipping will occur only in the smaller pulley since the angle of contact of the belt is less in the case of smaller pulley.

So, tension in the belt on the resisting side is to be calculated in respect of smaller pulley.

$$\log_{10} T_2 - \log_{10} T_1 = 0.434\mu_s\,\alpha.\quad T_2 = 2\,000\text{ N},\ \mu_s = 0.25,\ \alpha = 130°,\ T_1 = ?$$

$$\log_{10} 2\,000 - \log_{10} T_1 = 0.434 \times 0.25 \times 2.27$$

$$\log_{10} T_1 = 3.30 - 0.246 = 3.054 \quad\text{or}\quad T_1 = 1\,132.4\text{ N}$$

**Torque exerted by the belt on the bigger pulley**

Torque exerted $= (T_2 - T_1) \times r = (2\,000 - 1\,132.4) \times 0.3$ N.m = 260.28 N.m

| |
|---|
| When a belt connects two pulleys, slipping will occur first only in the pulley with which contact angle of the belt is less. |

**$\mu_s$ utilised in the bigger pulley when slipping occurs in the smaller pulley**

Contact angle of the bigger pulley = 230°

$$\log_{10} T_2 - \log_{10} T_1 = 0.434 \times \mu_s \times \alpha.$$

$T_2 = 2\,000$ N, $T_1 = 1\,132.4$, $\alpha = 230°$ (4 radians), $\mu_s = ?$

$$\log_{10} 2\,000 - \log_{10} 1\,132.4 = 0.434 \times \mu_s \times 4$$

$$\mu_s = \frac{3.30 - 3.054}{0.434 \times 4} = \frac{0.246}{0.434 \times 4} = 0.142$$

**Answers**

(i) Torque exerted by the belt on the bigger pulley = 260.28 N.m

(ii) Coefficient of friction of the bigger pulley utilised is only 0.142 as against 0.25.