26/9/13

2 copy

Roll No. | | | | | | | | | | |

B.E / B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, NOV / DEC 2013

COMMON TO ALL BRANCHES

54

Semester- 1

MA 131/MA 171/MA 9111 & MATHEMATICS-I

(Regulation 2002/2004/2008)

Time: 3 Hours | Answer ALL Questions | Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. Find the eigen values of the matrix $\begin{bmatrix} 1 & 7 & 8 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$ and also find the eigen values of $2A^{-1}$.

2. Write down the matrix of the quadratic form $2xy + 2yz + 2zx$.

3. Test the convergence of the series $6 - 10 + 4 + 6 - 10 + +6 - 10 + 4 + ...$

4. Discuss the convergence of the series $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + ...$

5. If $x = r\cos\theta, y = r\sin\theta$, prove that $J\,J' = 1$.

6. Expand $e^x \cos y$ in powers of x and y upto first degree.

7. Evaluate $\int_0^\infty x^4 e^{-x} dx$ using gamma function.

8. Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$.

9. Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} (x^2 + y^2)\, dx\, dy$

10. Evaluate $\int_1^a \int_1^b \frac{dxdy}{xy}$

Part – B (5 x 16 = 80 marks)

11. Verify Cayley-hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1} and A^4. (16)

12. a)

(i) Test for convergence the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + ... + \infty$ (8)

(ii) Test for convergence the series

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + ... + \frac{2^n - 2}{2^n + 1}x^{n-1} + ... (x > 0)$$ (8)

(OR)

b)

(i) State the values of for convergence of the series

$$\frac{x}{\sqrt{1}} - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + ... + \infty$$ (8)

(ii) Test for convergence the series

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + ... + \infty$$ (8)

13. a)

(i) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ (10)

(ii) If $u = xy + yz + zx$, where $x = e^t$, $y = e^{-t}$ and $z = \frac{1}{t}$, find $\frac{du}{dt}$. (6)

(OR)

b) Examine the function $f(x, y) = x^3 y^2 (12 - x - y)$ (16)

14. a)

(i) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (10)

(ii) Evaluate $I = \int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$. (6)

(OR)

b)

(i) Find the area of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$, using Gamma Functions. (8)

(ii) Evaluate $I = \int_0^{\infty} e^{-x^2} dx$ and prove $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (8)

Roll No. | | | | | | | | | | |

15. a)

(i) Change the order of integration $I = \int_0^a \int_y^a \frac{x}{x^2 + y^2}\, dx\, dy$ and then evaluate. (8)

(ii) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$ (8)

(OR)

b) (i) Evaluate $\iint x\, y\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = 1$. (8)

(ii) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{1 - x^2 - y^2 - z^2}}$. (8)