63

Reg No. | | | | | | | | | | |

ANNA UNIVERSITY, CHENNAI - 25

B.E./B.Tech. DEGREE END SEMESTER EXAMINATIONS- APRIL/MAY 2011

FIRST SEMESTER

MA 9111 – Mathematics I

(Common to all Branches)

Time: 3 Hours **Answer All Questions** **Max. Marks: 100**

Part – A **(10x2=20)**

1. For an orthogonal matrix A, prove that $|A| = \pm 1$.
2. Find the nature of the quadratic form
$$x_1^2 + 3x_2^2 + 4x_3^2 + 2x_2x_3 + 4x_1x_3 + 2x_1x_2$$
3. Define a bounded sequence.
4. Explain the convergence of Logarithmic series.
5. If $u = (x-y)(y-z)(z-x)$ then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
6. If $u = \frac{2x-y}{2}$ and $v = \frac{y}{2}$ then find $\frac{\partial(x,y)}{\partial(u,v)}$.
7. Evaluate $\int_0^\infty x^4 e^{-x} dx$ using gamma function.
8. Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$.
9. Evaluate $\int_2^3 \int_1^2 \frac{1}{xy} dx\, dy$.
10. Evaluate $\int_0^1 \int_0^2 \int_0^3 (xyz) dz dy dx$.

Part – B **(5 x 16 = 80)**

11. i). Test the convergence of the series using D' Alambert's ratio test.

$$1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\ldots \quad (x>0).$$ (8)

ii). State the values of x for which the following series converges.

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}\ldots+\infty.$$ (8)

12. a.i). Find the eigen values and eigen vectors for the given matrix

$$A=\begin{bmatrix}2&0&4\\0&6&0\\4&0&2\end{bmatrix}.$$ (8)

ii). Obtain the characteristic equation of the matrix $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$

and hence find A^4. (8)

(OR)

b.i). Reduce the quadratic form $x^2+5y^2+z^2+2xy+6xz+2yz$ to a canonical form through an orthogonal transformation. (10)

ii). Using Cayley – Hamilton theorem, find the inverse of the given

matrix $A=\begin{bmatrix}1&3&7\\4&2&3\\1&2&1\end{bmatrix}$ (6)

13.a.i). If $v = \log r$ *where* $r^2 = x^2 + y^2$ then find the value of

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}. \qquad (8)$$

ii). Obtain the dimensions of the rectangular box without a top of maximum capacity, given the surface area as 432 sq. meters. (8)

(OR)

b.i). Expand e^{xy} in powers of $(x-1)$ and $(y-1)$ up to the third degree terms by using Taylor's theorem. (8)

ii). Find the maximum and minimum values of the function $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$. (8)

14.a). Show that $\int_0^a \frac{\log(1+ax)}{1+x^2}\,dx = \frac{1}{2}\log(1+a^2)\tan^{-1}(a)$

and hence deduce that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log(2)$ using differentiation under the integral sign. (16)

(OR)

b.i). Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}}\,dx$ using gamma function (8)

ii). If $I = \int_0^\infty \frac{x^{n-1}}{1+x}\,dx = \frac{\pi}{\sin n\pi}$ then show that

$$\Gamma n \, \Gamma (1-n) = \frac{\pi}{\sin n\pi} \quad \textit{where} \quad 0 < n < 1. \qquad (8)$$

15.a.i). Evaluate $\int_{0}^{\pi}\int_{0}^{a\cos\theta}(r\sin\theta)\,dr\,d\theta.$ (8)

ii). Change the order of integration and evaluate

$$\int_{0}^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx.$$ (8)

(OR)

b.i). Find the area between the parabolas $y^2 = 4ax$ *and* $x^2 = 4ay$. (8)

ii). Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\,dx$ by changing into polar coordinates. (8)

&&&&&&&&&