

- (ii) Find the half-range Fourier Sine series expansion for the function $f(x) = x(\pi - x)$ in $0 < x < \pi$. Hence deduce the sum of the series $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots$ (8 Marks)

(OR)

- b(i) Find the half-range Fourier Cosine series expansion for the function $f(x) = x$ in $0 < x < l$. Hence deduce the sum of the series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ (8 Marks)

- (ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$, $-1 < x < 1$. (8 Marks)

- 13.a A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially displaced to the form $2 \sin\left(\frac{3\pi x}{l}\right) \cos\left(\frac{2\pi x}{l}\right)$ and then released. Find the displacement of the string at any distance x from one end at any time t . (16 Marks)

(OR)

- b A rod of 30 cm long has its ends $A = 0$ and $B = 30$ kept at $20^{\circ}C$ and $80^{\circ}C$ respectively, until steady state conditions prevail. The temperature at each end is then suddenly changed to $80^{\circ}C$ and kept so. Find the resulting temperature function $u(x, t)$. (16 Marks)

- 14.a(i) Find the Fourier transform of $f(x) = e^{-x^2/2}$. (8 Marks)

- (ii) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a > 0. \end{cases}$ Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. (8 Marks)

(OR)

- b Find the Fourier Sine and Fourier Cosine transforms of $f(x) = e^{-ax}$. Hence, using Parseval's identity, evaluate $\int_0^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$. (16 Marks)

- 15a.(i) Solve the equation $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$ given $y_0 = y_1 = 0$. (8 Marks)

- (ii) Use convolution theorem to find the inverse Z -transform of $\frac{z^2}{(z-1)(z-3)}$. (8 Marks)

(OR)

- b.(i) Evaluate $Z^{-1}[(z-5)^{-3}]$ for $|z| > 5$. (8 Marks)

- (ii) Derive a difference equation by eliminating the constants from $y_n = (A + Bn)3^n$. (8 Marks)

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