

Roll No.

--	--	--	--	--	--	--	--	--	--

**B.E / B.Tech ( Full Time ) DEGREE END SEMESTER EXAMINATIONS, APRIL / MAY 2014**

Common to All Branches

II Semester

**MA8251 MATHEMATICS-II**

(Regulation 2012)

Time: 3 Hours

Answer ALL Questions

Max. Marks 100

**PART-A (10 x 2 = 20 Marks)**

1. Solve  $(D^3 - 2D^2 - D + 2)y = 0$ ,  $D \equiv \frac{d}{dt}$ .
2. Find the Wronskian of the functions  $\cos 2x$  &  $\sin 2x$  and hence state whether they are linearly independent or not.
3. Find the unit vector normal to the surface  $x + 2y - z = 2$  at  $(1, 1, 1)$ .
4. Using Gauss' divergence theorem, show that  $\iint_S \vec{r} \cdot \hat{N} dS = 3V$ , where  $V$  is the volume of space enclosed by the surface  $S$ ,  $\hat{N}$  is the unit outward drawn normal to the surface and  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ .
5. Show that  $e^z$  is an entire function.
6. Determine the image of the circle  $|z| = a$  under the transformation  $w = 2z$ .
7. Find the residue of  $\frac{e^z}{z^5}$  at  $z = 0$ .
8. Classify the singularity of  $\sin\left(\frac{1}{1-z}\right)$ .

9. If  $F(s) = \frac{1}{s(s+a)}$  is the Laplace transform of a function  $f(t)$ , then find the value of  $\lim_{t \rightarrow \infty} [f(t)]$  without finding the function  $f(t)$ .
10. Use the first shifting property to find the inverse Laplace transform of  $\frac{1}{(s+3)^4}$ .

**Part – B ( 5 x 16 = 80 Marks)**

11. (i) Show that  $u = \log \sqrt{x^2 + y^2}$  is harmonic. Using Milne-Thompson method, find the analytic function  $f(z) = u + iv$ . (8)

(ii) Find the bilinear transformation which maps the points  $z = 0, -1, i$  onto the points  $w = i, 0, \infty$ . Under this mapping, find the image of the unit circle  $|z| = 1$ . (8)

12. a. (i) By the method of variation of parameters, solve the differential equation  $(D^2 - 6D + 9)y = \frac{e^{3t}}{t^2}$ . (8)

(ii) Solve:  $(D - 7)x + y = 0$ ;  $-2x + (D - 5)y = 0$ . (8)

(OR)

- b. (i) By the method of undetermined coefficients, solve the differential equation  $(D^2 - 2D)y = 5e^t \cos t$ . (8)

(ii) Solve:  $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{12 \log x}{x^2}$ . (8)

13. a. (i) Find the constants  $a, b, c$  such that the vector field given by  $\vec{F} = (x + 2y + az)\hat{i} + (bx + 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$  is irrotational. For this set of values of  $a, b, c$ , find the scalar potential of  $\vec{F}$ . (8)

(ii) Using Stokes' theorem, evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$  and C is the boundary of the triangle with vertices (0,0,0), (1,0,0) and (1,1,0). (8)

(OR)

b. (i) Prove that  $\nabla^2 [f(r)] = f''(r) + \frac{2}{r} f'(r)$  where  $r = |\vec{r}|$ ,  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ . (8)

(ii) Apply Green's theorem to prove that the area enclosed by a plane curve is  $\frac{1}{2} \int_C (x dy - y dx)$ . Hence find the area of a circle of radius 'a'. (8)

14. a. (i) Using Cauchy Residue theorem, evaluate  $\int_C \frac{\tan z}{(z^2 - 1)} dz$ , where C is the circle  $|z| = \frac{3}{2}$ . (8)

(ii) Expand  $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$  in a Laurent's series in  $|z| > 3$ . (8)

(OR)

b. (i) Using contour integration, show that  $\int_0^{2\pi} \frac{d\theta}{5 - 4 \sin \theta} = \frac{2\pi}{3}$ . (8)

(ii) Using Cauchy Integral formula, find the values of  $F(i)$  &  $F'(-1)$  given that

$F(\xi) = \oint_C \frac{4z^2 + z + 5}{z - \xi} dz$ , where C is the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ . (8)

15. a. (i) Use Laplace transform method to solve the differential equation  $y'' + y = 2$  given that  $y(0) = 0, y'(0) = 1$ . (8)

(ii) Find the Laplace transform of the square wave function

$$f(t) = \begin{cases} 1, & 0 < t < \frac{a}{2} \\ -1, & \frac{a}{2} < t < a \end{cases}, \text{ where } f(t+a) = f(t). \quad (8)$$

(OR)

b. (i) Using Laplace transform, evaluate  $\int_0^{\infty} \frac{\sin^2 t}{te^t} dt.$  (8)

(ii) Use convolution theorem to find the inverse Laplace transform of  $\frac{1}{s(s+a)^3}.$  (8)