Roll No | | | | | | | | | | |

**B.E. / B.TECH. (Full Time) DEGREE END SEMESTER EXAMINATIONS, APRIL / MAY, 2014**

COMMON TO ALL BRANCHES

First Semester

**MA 8151 Mathematics - I**
(Regulation 2012)

**Time :** 3 Hours — Answer **ALL** Questions — **Max. Marks:** 100

**PART-A (10 x 2 = 20 Marks)**

1. Find the eigenvalue(s) of an idempotent matrix A.

2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 0 & 5 & 2 \end{pmatrix}$. Find the sum and product of the eigenvalues of $A^{-1}$.

3. Find the nature of the series $\sum_{n=1}^{\infty} \frac{1}{n!}$.

4. Test whether convergent series is absolutely convergent? Justify the claim.

5. Find $\partial u/\partial x$ and $\partial u/\partial y$ when $u(x,y) = x^y + y^x$.

6. If $z = f\left(\frac{y}{x}\right) + g\left(\frac{x}{y}\right)$, then find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$, using Euler's theorem.

7. Evaluate the improper integral $\int_{-2}^{2} \frac{1}{x^2} dx$.

8. Evaluate $\int_{0}^{\pi/2} \sqrt{\cot\theta}\, d\theta$, using Beta and Gamma functions.

9. Find the area of a circle of radius 'a' by double integration in polar coordinates.

10. Evaluate $\int_{x=0}^{1} \int_{y=0}^{2} \int_{z=1}^{2} xy \, dx\, dy\, dz$.

**Part – B ( 5 x 16 = 80 Marks)**

11.(i) Evaluate $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$, where $a, b > 0$, using Leibnitz's rule. (6)

11.(ii) Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ and hence find the value of $\Gamma(1/2)$. (10)

□

12a.(i) Diagonalise the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$ by means of an orthogonal transformation. (8)

12a.(ii) Verify that the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ satisfies its own characteristic equation and hence compute the elements of the matrix given by $A^4 - 5A^3 + 8A^2 - 2A + I$. (8)

**(OR)**

12b. Reduce the quadratic form $x^2 + 2y^2 + z^2 - 2xy + 2yz$ to the canonical form by an orthogonal transformation and hence find the index, signature and nature of the quadratic form, and also find a non-zero set of values of x, y, z which make this quadratic form zero. (16)

□

13a.(i) Test for the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots$ to $\infty$ by D'Alembert's ratio test. (8)

13a.(ii) Discuss the nature of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$, where $p > 0$, by using Integral test. (8)

**(OR)**

13b.(i) Is the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + .......$ to $\infty$ conditionally convergent or absolutely convergent? Justify the claim. (8)

13b.(ii) For what values of x, the series $\frac{1}{(1-x)} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + ...,$ to $\infty$ converges? (8)

□

14a.(i) Let $u = 3x + 2y - z, v = x - 2y + z$ and $w = x(x + 2y - z)$. Are u, v and w functionally related? . If so, find this relationship. (8)

14a.(ii) Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ up to the second degree terms. (8)

**(OR)**

14b.(i) If $z=f(x,y)$, where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, then show that

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}.$$ (8)

14b.(ii) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area 432 square meters. (8)

□

15a.(i) Evaluate integral $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ by changing the order of integration. (8)

15a.(ii) Evaluate $\iiint_V xyz \, dx \, dy \, dz$, where V is the volume of the positive octant of the sphere $x^2 + y^2 + z^2 = 1$ by transforming to spherical polar coordinates. (8)

**(OR)**

15b.(i) Evaluate $\iint_D xy\sqrt{(1-x-y)} \, dx \, dy$, where D is the region bounded by x = 0, y = 0 and $x + y = 1$, using the transformation $x + y = u$, $y = uv$. (8)

15b.(ii) Find the area of the portion of the sphere $x^2 + y^2 + z^2 = 3^2$ lying inside the cylinder $x^2 + y^2 = 3y$. (8)

□

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