Roll No. | | | | | | | | | | |

B.E / B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, APRIL / MAY 2014

Common to All Branches

I SEMESTER

MA9111 MATHEMATICS-I

(Regulation 2008)

Time: 3 Hours | Answer ALL Questions | Max. Marks 100

Part - A (10 × 2 = 20)

1. If two of the eigenvalues of a matrix $(A)_{3\times3}$ are equal and double the third and if trace(A) is 5, then find all the eigenvalues of A.
2. Find the rank, index, signature and nature of the quadratic form whose canonical form is $0u^2+3v^2+3w^2$.
3. Using integral test, show that $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.
4. Examine the convergence of the series: $1+\frac{1}{4^{2/3}}+\frac{1}{9^{2/3}}+\frac{1}{16^{2/3}}+\cdots$.
5. If $z=\frac{x^2+y^2}{\sqrt{x+y}}$, then find $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$.
6. Find the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ of the transformation $x=r\cos\theta,\ y=r\sin\theta$.
7. Compute $\beta\left(\frac{5}{2},\frac{3}{2}\right)$.

8. Evaluate the second kind improper integral $\int_{0}^{\pi/2} \tan x dx$.

9. Sketch the region of integration of the double integral $\int_{0}^{\pi} \int_{2\sin\theta}^{4\sin\theta} f(r,\theta) dr d\theta$.

10. Express the volume of a solid enclosed by a surface $z = f(x, y)$ as a double integral.

Part - B (5 × 16 = 80)

11 (i) Use Cayley Hamilton theorem to find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$. (8)

(ii) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to a canonical form by orthogonal transformation. (8)

12a. (i) Using comparison test, check for convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \cdots$. (8)

(ii) Find the interval of convergence of the logarithmic series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ (8)

(OR)

b. (i) Test for absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$. (8)

(ii) Discuss the convergence of the series $x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \cdots$. (8)

13a. (i) Find the maxima and minima of $xy(a - x - y),\ a > 0$. (10)

(ii) If $z = e^{ax+by} f(ax - by)$, then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. (6)

(OR)

b. (i) Expand $e^x \log(1+y)$ about (0,0) upto the third degree terms. (10)

(ii) If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If yes, find the relation between them. (6)

14a. (i) Express the integral $\int_0^\infty e^{-ax} x^{m-1} \sin bx dx$ in terms of gamma functions.

(10)

(ii) Prove that $\frac{d}{dx}[erfc(ax)] = -\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$. (6)

(OR)

b. (i) Using Leibnitz's rule, evaluate $\int_0^1 x^m (\log x)^n dx$. (10)

(ii) Prove that $\beta(m, \mathrm{n}) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (6)

15a. (i) Change the order of integration and hence evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dx dy$ (8)

(ii) Using triple integral, find the volume of the solid bounded by the planes $x = 0, y = 0, z = 0 \;\&\; x + y + z = 1$. (8)

(OR)

b. (i) Using double integral, find the area between the parabola $y^2 = 4x - x^2$ and the line $y = x$.

(8)

(ii) By changing into polar co-ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$. (8)