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B.E / B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, NOV / DEC 2013

COMPUTER SCIENCE AND ENGINEERING

Fifth Semester

CS 9302 – THEORY OF COMPUTATION

(Regulation 2008)

Time : 3 Hours | Answer ALL Questions | Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. Prove by mathematical induction that $n^4 - 4n^2$ is divisible by 3 for $n \geq 0$.
2. Give an ε -NFA that will accept 0*1*. Convert the ε -NFA to DFA.
3. Write a regular expression that recognizes the set of all strings that do not contain the substrings 00 and 11 over the alphabet (0+1)*.
4. Prove that the union of two regular languages is also regular.
5. Write a context free grammar for the language L = $\{w\overline{w}^R \mid w \in (a+b)^*\}$ ($\overline{w}^R$ is the reverse of the complement of string w).
6. Consider the following grammar

   $S \rightarrow AS \mid \varepsilon$

   $A \rightarrow aa \mid ab \mid ba \mid bb$

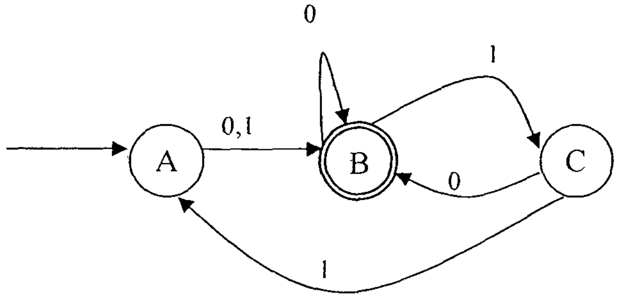
   Give the leftmost derivation for the string *baabab*.
7. Design a Turing machine that will reverse the binary string given as input.
8. If L is a context free language and R is a regular language, then prove that L - R is a context free language.
9. Prove that if a language L and its complement are recursively enumerable, then L is recursive.
10. State Rice's theorem.

Part – B (5 x 16 = 80 marks)

11. (i) Prove that a language L is accepted by some ε -NFA if and only if L is accepted by some DFA. (8)

    (ii) Construct a DFA that will accept the set of all strings which when interpreted as a binary integer is a multiple of 4. (8)

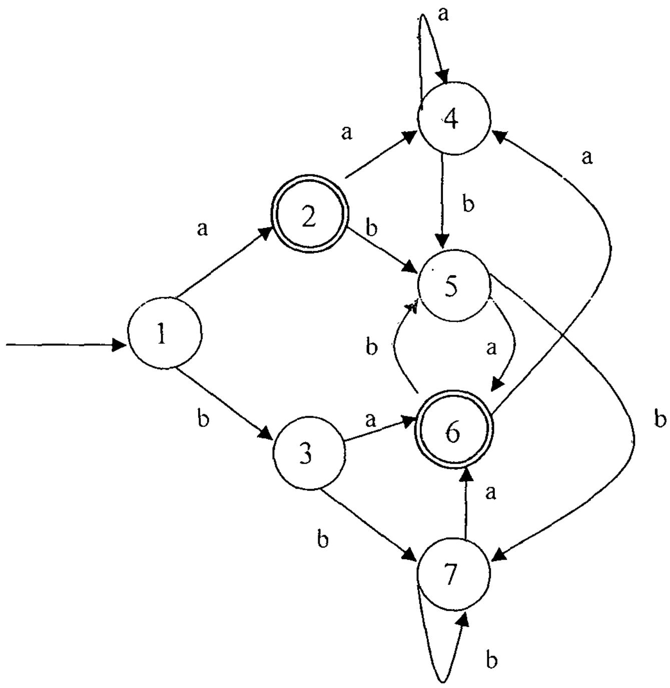
12. a) (i) Find the regular expression corresponding to the DFA given below using Kleene's method. (10)



(ii) Draw DFA A_1 that accepts strings that end with a 1 and DFA A_2 that accepts strings that end with a 0 over the alphabet (0+1)*. Find the DFA that accepts the intersection of the languages accepted by A_1 and A_2. (6)

OR

b) (i) Minimize the following automaton using the table filling algorithm. (10)



(ii) Using pumping lemma prove that the language L = $\{0^i 1^j 0^k \mid k>i+j\}$ is not regular. (6)

13. a) (i) Let $G = (V, T, P, S)$ be a context free grammar. Suppose there is a parse tree with root labeled by variable A and with yield w, where w is in T^*, then prove that there is a leftmost derivation $A \overset{*}{\underset{lm}{\Rightarrow}} w$ in grammar G. (8)

(ii) Design a push down automaton (PDA) that will accept strings in which the sum of the number of a's and the number of b's is equal to the number of c's over the alphabet (a+b+c)*. (8)

OR

b) (i) If $L = L(P_F)$ for some PDA $P_F = (Q, \Sigma, \Gamma, \delta_F, q_0, Z_0, F)$ that accepts by final state, then prove that there is a PDA P_N that accepts by empty stack such that $L = N(P_N)$. (8)

(ii) Find a context free grammar that generates $L(M)$ where the PDA M is given by

$M = (\{q_0, q_1, q_2\}, \{a,b\}, \{Z_0, A\}, \delta, q_0, Z_0, \{q_2\})$, where δ is given by

$\delta(q_0, a, Z_0) = \{(q_1, AZ_0)\}$
$\delta(q_0, a, A) = \{(q_1, AA)\}$
$\delta(q_1, a, A) = \{(q_0, AA)\}$
$\delta(q_1, \varepsilon, A) = \{(q_2, AA)\}$
$\delta(q_2, b, A) = \{(q_2, \varepsilon)\}$

(8)

14. a) (i) Simplify the following grammar and convert to Chomsky normal form. (8)

$S \rightarrow ASB \mid \varepsilon$
$A \rightarrow aAS \mid a$
$B \rightarrow SbS \mid A \mid bb$

(ii) Construct a Turing machine to accept the language $\{a^i b^j \mid i<j\}$. (8)

OR

b) (i) Convert the following grammar to Greibach normal form. (8)

$S \rightarrow XA \mid BB$
$B \rightarrow b \mid SB$
$X \rightarrow b$
$A \rightarrow a$

(ii) Construct a Turing machine that will compute the quotient when m is divided by n (m/n). m and n are placed on the tape in unary and after computation, the quotient should be available on the tape in unary. (8)

15. a) Define the languages L_u, L_{ne}. Prove that L_u and L_{ne} are recursively enumerable.

OR

b) Consider the following Turing machine $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, \delta, q_1, B, q_3)$ where δ is given as

$\delta(q_1,0) = (q_1, 0, R)$
$\delta(q_1,1) = (q_1, 1, R)$
$\delta(q_1,B) = (q_2, B, L)$
$\delta(q_2,0) = (q_1, 0, R)$
$\delta(q_2,1) = (q_3, B, R)$
and input string $w = 001$.

Construct a Modified Post's Correspondence Problem (MPCP) instance from the Turing machine M. Check if the MPCP instance has a solution.