

Time: 3 Hours

Answer ALL Questions

Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. If A and B are events in S such that $P(A) = 1/3$, $P(B) = 1/4$, $P(A \cup B) = 1/2$. Find $P(A \cap \bar{B})$ and $P(A/\bar{B})$.
2. The distribution function of a random variable is given by $F(x) = 1 - (1+x)e^{-x}$, $x \geq 0$. Find the density function and mean.
3. An experiment succeeds twice as many times as it fails. Find the chance that in 6 trials, there will be at least 5 successes.
4. Find the mean and variance of uniform distribution.
5. The joint probability mass function is given by $p(x, y) = k(2x + 3y)$, $x = 0, 1, 2$, $y = 1, 2, 3$. Find the value of k .
6. The joint probability density function of the random variable is given by $f(x, y) = kxye^{-(x^2+y^2)}$, $x > 0, y > 0$. Find the marginal density function of X.
7. Define Type I and Type II Error.
8. Explain any two uses of chi-square distribution.
9. Write the basic assumptions in analysis of variance.
10. Describe the Latin Square Design.

Part B $(10 \times \overset{8}{2} = 20)$ 20.

11. (i) Test whether the sample having the values 63, 63, 64, 55, 66, 69, 70, 70 and 71 has been chosen from a population with mean of 65 at 5% level of significance. (8)
- (ii) In a sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 94.5 and in another sample of 10 observations, it was found to be 101.7. Test whether the difference of variances is significant. (8)
12. (a) (i) If at least 1 child in a family with 2 children is a boy, what is the probability that both children are boys? (6)
- (ii) A random variable X has the following probability distribution.
x: -2 -1 0 1 2 3
P(x): 0.1 k 0.2 2k 0.3 3k
Find k.
(a) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$.
(b) Find the cdf of X.
(c) Find the mean of X. (10)

(OR)

(b)(i) A man has five coins, one of which has two heads. He randomly takes out a coin and tosses it three times: What is the probability that it will fall head upward all the times. (8)

(ii) The bag A contains 5 red and 3 green balls and bag B contains 3 red and 5 green balls. One ball is drawn from bag A and two from bag B. Find the probability that of the three balls drawn two are red and one is green. (8)

13. (a) (i) Define Binomial distribution and obtain its moment generating function. From the moment generating function, find the mean and variance. (8)

(ii) A coin is tossed four times what is the probability of getting

(a) no head (b) exactly one head (c) at least two heads (d) more than two heads. (8)

(OR)

(b) (i) Find the moment generating function of Gamma distribution and hence find its mean and variance. (8)

(ii) Out of 800 families with 4 children each, how many families would be expected to have

(i) 2 boys and 2 girls (ii) at most 2 girls and (iii) at least one boy (iv) Children both of sexes? Assume equal probability for boys and girls. (8)

14. (a) (i) The RVs X and Y have the joint pdf

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional pdf of X given Y and Y given X. (8)

(ii) The joint density function of random variables X and Y is given by

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the correlation co-efficient between x and y. (8)

(OR)

b) (i) Two random variables X and Y have the following joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find $E[Y/X=x]$, $E[XY/X=x]$ and $\text{Var}[Y/X=x]$. (8)

(ii) If X and Y are independent random variables, then prove that $E[Y/X]=E[Y]$. (8)

15. (a) A call Taxi agency uses five different brands of tyres in the process of deciding the brand of tyres to buy as a standard equipment for its fleet, finds that each of five tyres of each brand had the life of the following number of kilometers. (16)

A	41000	35000	45000	46000	36000
B	39000	36000	37000	39000	42000
C	42000	37000	35000	39000	37000
D	35000	45000	43000	37000	40000
E	38000	39000	30000	42000	43000

Test the hypothesis that the five brands of tyres have the same average life.

(OR)

- (b) The following table gives per hectare yield for three varieties of wheat each grown on five plots:

Plot of land	Variety of Wheat		
	A	B	C
1	5	3	10
2	6	5	13
3	8	2	7
4	1	10	13
5	5	0	17

Test at 5% level of significance.

(16)

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