12/11/13 (32)

Reg. No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

B.E./ B. Tech, DEGREE END SEMESTER EXAMINATION, Nov / Dec - 2013

MECHANICAL ENGINEERING

THIRD SEMESTER

MA 8302 – PARTIAL DIFFERENTIAL EQUATIONS

(Regulation – 2012)

Time: 3 hours **Maximum: 100 Marks**

Answer ALL Questions

Part – A ($10\times 2=20$ **marks)**

1. Find the complete solution of $\sqrt{p}+\sqrt{q}=1$.
2. Find the general solution of $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial z}{\partial y}-z=0$.
3. State Dirichlet's conditions for existence of Fourier series.
4. Compute the constant term of the Fourier series of $f(x)$, if $f(x)=|\cos x|$ in $-\pi<x<\pi$.
5. What are the possible solutions of the one dimensional heat flow equation?
6. A bar 30cm, long, with insulated lateral surface, has its ends A and B kept at 20° and 80°, respectively, find the steady temperature of the bar, by taking A at $x=0$.
7. Write the second derivative using the second order difference.
8. Write the Crank-Nicholson difference scheme to solve $u_{xx}=au_t$.
9. Write the iterative formula for the solution of system of linear equations in SOR method.
10. Obtain the difference formula for $u_{xx}+u_{yy}=0$.

Part – B ($5\times 16=80$ marks)

11. i) Find the Fourier series expansion of $f(x)=x^2$ in $-\pi<x<\pi$. Hence deduce for $\sum_{n=1}^{\infty}\frac{1}{n^2}$. (8)

ii) Obtain the Fourier cosine series expansion of $f(x)=x$ in $0<x<l$. Hence deduce the value of $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+........to\ \infty$. (8)

12. (a) i) Find the partial differential equation by eliminating the arbitrary functions f and g from $z(x,y)=xf(y)+yg(x)$. (8)

ii) Find the general solution of $z(x-y)=px^2-qy^2$. (8)

(OR)

(b) i) Find the singular solution of $z=px+qy+\sqrt{1+p^2+q^2}$. (8)

ii) Find the general solution of $(D^2-2DD'+D'^2)z=\sin(x-2y)+x^2y^2$. (8)

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l - x)$, find the displacement of the string in the subsequent time. (16)

(OR)

(b) A rectangular plate with insulated surface is bounded by the lines $x = 0, x = a, y = 0$ and $y = b$. The temperature along the edge $y = b$ kept at $100°C$. While the temperature along the other three edges are at $0°C$. Find the steady- state temperature at any point in the plate. (16)

14. (a) i) Solve the following system of linear equations by using Gauss-Elimination method:

$3x + 4y + 5z = 18, 2x - y + 8z = 13, 5x - 2y + 7z = 20$. (8)

ii) Using Crank-Nicholson's scheme, solve $u_{xx} = 16u_t, 0 < x < 1, t > 0$ given $u(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = 100t$. Compute $u(x,t)$ for two step in t direction taking $h = \frac{1}{4}$. (8)

(OR)

(b) i) Solve the following tri-diagonal system of equation using Thomas algorithm:

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -2 \end{bmatrix}$$ (8)

ii) Find the solution of the two dimensional heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ subject to the initial condition $u(x, y, 0) = \sin \pi x \sin \pi y$, $0 \le x, y \le 1$ and the boundary conditions $u = 0$ on the boundary for $t \ge 0$, using Peaceman-Rachford ADI method by taking $h = \frac{1}{3}$, $\lambda = \frac{1}{6}$ and integrate for one time step. (8)

15. (a) i) Solve the following system of linear equations by Gauss-Seidel method correct to three decimal places: $x + y + 54z = 110, 27x + 6y - z = 85, 6x + 15y + 2z = 72$. (8)

ii) Solve $u_{xx} + u_{yy} = 0$ over the square mesh with 1 unit of sub-square of side 4 units, satisfying the following boundary conditions correct to 2 decimal places:

(I) $u(0, y) = 0$, for $0 \le y \le 4$, (II) $u(4, y) = 12 + y$, for $0 \le y \le 4$,

(III) $u(x,0) = 3x$, for $0 \le x \le 4$, (IV) $u(x,4) = x^2$, for $0 \le x \le 4$. (8)

(OR)

(b) i) Using Gauss-Jacobi method, solve the system of equations three decimal places:

$28x + 4y - z = 32, 2x + 17y + 4z = 35, x + 3y + 10z = 24$. (8)

ii) Solve the Poisson's equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary with 1 unit length on the mesh correct to three decimal places. (8)

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