Registration number | | | | | | | | | ✓

**B.E./B.Tech (Full Time) Degree End Semester Examinations, November 2013**
**Third Semester**
**MA8356 PROBABILITY AND STATISTICS**
**Common to Industrial, Manufacturing, Rubber and Production, Food and Pharma and Chemical Engineering**
(Regulation 2012)
**(Use of Statistical tables and calculators may be permitted)**

**Max Marks: 100** **ANSWER ALL QUESTIONS** **Time: 3 Hours**

## PART A (10 x 2 = 20)

1. A random variable $X$ has the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $F(x)$ and use it to evaluate $P(0 < X \leq 1)$.

2. Suppose the moment generating function of a random variable $X$ is $e^t(5 - 4e^t)^{-1}$, find $P(X = 3)$

3. The lines of regression of $X$ on $Y$and that of $Y$ on $X$ are given by $3X + Y = 10$ and $3X + 4Y = 12$. Determine the means of $X$ and $Y$.

4. State Cental Limit Theorem.

5. Using 0.01 level of significance, test the null hypothesis that $\sigma = 2$ if a random sample of size $n = 31$ yields a sample variance of 1.8.

6. Test whether the following arrangements of defective, $d$ and non-defective, $n$ items produced by a certain machine is random at 0.01 level of significance:
n n n n d d d d n n n n n n n n n d d n n d d d d

7. Present the ANOVA table for the one way classification.

8. Describe $2^2$ factorial design.

9. Present the control chart values for the number of defects chart.

10. To check the strength of carbon steel for use in chain links, the yield stress of a random sample of 25 pieces was measured yielding a mean and standard deviation of 52,800 psi and 4600 psi respectively. Establish the tolerance limits with $\alpha = 0.05$ and $P = 0.99$.

## PART B (5 x 16 = 80)

11. (a) i. The milk bottling company has decided to employ a control chart to provide continuous monitoring of the weight of its milk containers. A random sample of $n = 5$ milk containers is selected every 20 minutes and their weights are measured as shown below:

| Sample Number | | Weight | per | container | |
|---|---|---|---|---|---|
| 1 | 2.106 | 1.965 | 2.081 | 2.079 | 2.129 |
| 2 | 1.950 | 1.994 | 2.058 | 2.039 | 2.080 |
| 3 | 2.095 | 2.043 | 2.158 | 2.005 | 2.085 |
| 4 | 2.088 | 2.053 | 2.025 | 2.011 | 2.128 |
| 5 | 1.980 | 2.080 | 2.189 | 2.024 | 2.072 |

Use $\bar{X}$ and $R$ chart to comment on the statistical control of the process.

(8 Marks)

ii. Consider the data below on the number of defective electronic components in samples of size 50. Twenty samples were taken in order to establish preliminary control chart values

| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of Defectives | 8 | 6 | 5 | 7 | 2 | 5 | 3 | 8 | 4 | 4 |
| Sample | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Number of Defectives | 3 | 1 | 5 | 4 | 4 | 2 | 3 | 5 | 6 | 3 |

Test the control of the process using control chart for fraction defectives.

(8 Marks)

12. (a) i. The probability of a man hitting the target is 1/4.

A. If he fires 7 times, what is the probability of his hitting the target at least twice?

B. How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3?

(8 Marks)

ii. Let $X$ denote the weights of parcels which can be modelled as $N(5, 16)$ normal random variable. Find $P(1 < X < 10)$ and $P(X > 9)$. (8 Marks)

**[OR]**

(b) i. The number of accidents in a year to taxi drivers in a city follows a poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents in a year (ii) more than 3 accidents in a year. (8 Marks)

ii. Find the probability density function of the random variable $Y = X^2$, where $X$ is a standard normal random variable. (8 Marks)

13. (a) i. The joint probability density function(pdf) of $(X, Y)$ is given by

$$f(x,y) = \begin{cases} (1/4)(1+xy), & -1 < x < 1, -1 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Are $X$ and $Y$independent? (8 Marks)

ii. Suppose $X$ and $Y$ are independent random variables following exponential distribution with parameter 1, determine the probability density function of the random variable $\frac{X}{Y}$. (8 Marks)

[OR]

(b) i. The joint probability density function of $(X, Y)$ is given by

$$f(x,y) = \begin{cases} 2-x-y & 0<x<1, 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

Find the correlation coefficient between $X$ and $Y$. (10 Marks)

ii. Given the joint probability density function of $(X, Y)$ as

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0<x<1, 0<y<2 \\ 0, & \text{otherwise} \end{cases}$$

determine the conditional density functions. (6 Marks)

14. (a) i. An examination was given to two classes consisting of 40 and 50 students respectively. In the first class, the mean grade was 74 with a standard deviation of 8, while in the second class the mean grade was 78 with a standard deviation of 7. Test whether $\mu_1 - \mu_2 = 0$ at 1% level of significance.

(8 Marks)

ii. Five coins are tossed 320 times. the number of heads observed is given below:

| Number of heads | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 15 | 45 | 85 | 95 | 60 | 20 |

Examine whether the coin is unbiased. Use 5% level of significance.

(8 Marks)

[OR]

(b) i. Two independent samples of sizes 7 and 6 have the following values

| Sample A | 28 | 30 | 32 | 33 | 31 | 29 | 34 |
|---|---|---|---|---|---|---|---|
| Sample B | 29 | 30 | 30 | 24 | 27 | 28 | |

Examine whether the samples have been drawn from normal population having the same variance.

(8 Marks)

ii. The table below shows the relation between performance of students in mathematics and physics. Test the hypothesis that performance in physics is independent of performance in mathematics using 1% level of significance

| | High grades | Medium grades | Low grades |
|---|---|---|---|
| High grades | 56 | 71 | 12 |
| Medium grades | 47 | 163 | 38 |
| Low grades | 14 | 42 | 85 |

(8 Marks)

15. (a) The following table shows the yield per acre for 4 different plant crops grown on lots treated with 3 different types of fertilizers. Determine whether there is a difference in yield per acre i) due to fertilizers ii) due to crops, at 0.05 level of significance.

| | Crop I | Crop II | Crop III | Crop IV |
|---|---|---|---|---|
| Fertilizer A | 4.5 | 6.4 | 7.2 | 6.7 |
| Fertilizer B | 8.8 | 7.8 | 9.6 | 7.0 |
| Fertilizer C | 5.9 | 6.8 | 5.7 | 5.2 |

(16 Marks)

**[OR]**

(b) A farmer wishes to test the effects of four different fertilizers A,B,C,D on the yield of wheat. In order to eliminate sources of error due to variability in soil fertility, he uses the fertilizers in a Latin-square arrangement,as shown in the following table, where the numbers indicate yields in bushels per unit area. Perform an analysis of variance to determine whether there is a difference between the fertilizers at 0.05 level of significance.

| | | | |
|---|---|---|---|
| A 18 | C 21 | D 25 | B 11 |
| D 22 | B 12 | A 15 | C 19 |
| B 15 | A 20 | C 23 | D 24 |
| C 22 | D 21 | B 10 | A 17 |

(16 Marks)

&&&&&&&& ***GOOD LUCK*** &&&&&&&&