ANNA UNIVERSITY – UNIVERSITY DEPARTMENT
B.E. (FULL-TIME) DEGREE EXAMINATION, MAY 2013

Sixth Semester – R2008
Branch: Computer Science & Engineering

CS9032 – GRAPH THEORY
(End-Semester Examination)

Time: Three Hours                Max. Marks: 100

Answer **ALL** Questions

**Part A** ($10 \times 2 = 20$ Marks)

1. Define ring-sum of two graphs.
2. Give an example of an Euler graph which is arbitrarily traceable.
3. State any two properties of a graph with $n^2$ edges, where $n$ is the number of vertices in the graph.
4. Define eccentricity of a graph.
5. What is edge connectivity?
6. Give an example of 1-isomorphic but not isomorphic graphs.
7. What is the rank of a circuit matrix if $e$ and $n$ are respectively the number of edges and vertices of $G$.
8. Suppose $X$ is the adjacency matrix of a graph. What does $X^k$ denote?
9. What is the upper limit of the chromatic number?
10. In a directed graph, when do we say a vertex is isolated?

**Part B** ($5 \times 16 = 80$ Marks)

11. Prove the following.
    (a) If a graph has exactly two vertices of odd degree, there must be a path joining these two vertices. (4)
    (b) A connected graph is an Euler graph if and only if every vertex has even degree. (6)
    (c) A connected graph is an Euler graph if and only if it can be decomposed into circuits. (6)

12. (a) Prove the following.

i. Every tree has either one or two centers. (4)

ii. A graph is a tree if and only if it is minimally connected. (4)

iii. Number of vertices in a binary tree is always odd. (4)

iv. Number of pendent vertices in a binary tree is $(n+1)/2$. (4)

(OR)

(b) i. The ring-sum of any two cut-sets in a graph is either third cut-set or an edge disjoint union of cut-sets. (8)

ii. Two graphs are 2-isomorphic if and only if they have circuit correspondence. (4)

iii. A vertex $v$ in a connected graph $G$ is a cut-vertex if and only if there exists two vertices $x$ and $y$ in $G$ such that every path between $x$ and $y$ passes through $v$. (4)

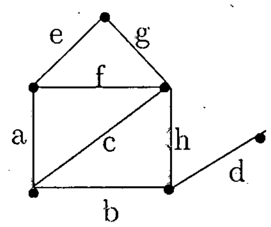
13. (a) Prove that a graph is non-planar if and only if it contains a subgraph homeomorphic to $K_5$ or $K_{3,3}$. (16)

(OR)

(b) i. If $A(G)$ is an incidence matrix of a connected graph $G$ with $n$ vertices, then prove that rank of $A(G)$ is $n-1$. (6)

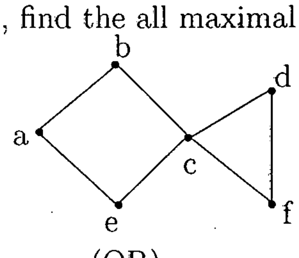
ii. Prove that the reduced incidence matrix of a tree is non-singular. (6)

iii. Generate the circuit matrix for the following graph. (4)




14. (a) i. If the edges of a connected graph are arranged in the same order for the columns of the incidence matrix $A$ and the path matrix $P(x,y)$, then prove that the product (mod 2) $A.P^T(x,y) = M$, where the matrix $M$ has 1's in two rows $x$ and $y$, and the rest of the $n-2$ rows are all zeros. (8)

ii. For the following graph, find the all maximal independent sets. (8)




(OR)

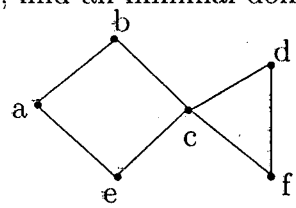
(b) i. Prove that the chromatic polynomial of a tree with $n$ vertices is (4)

$$\lambda(\lambda-1)^{n-1}$$

ii. Prove that the chromatic polynomial of a complete graph is (4)

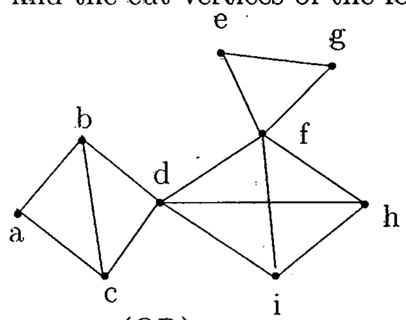
$$\Pi_{i=0}^{n-1}(\lambda-i)$$

iii. For the following graph, find all minimal dominating sets. (8)



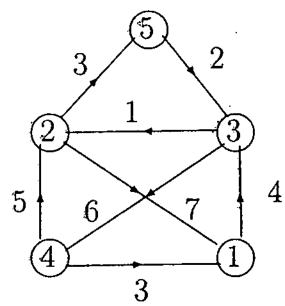

15. (a) i. Describe the algorithm for finding cut-vertices of a graph. (10)

ii. Using this algorithm, find the cut-vertices of the following graph. (6)




(OR)

(b) i. Find the distance matrix for the all pair shortest path of the following graph. (10)




ii. Derive the shortest path from $a$ to $b$ by generating appropriate matrix from the distance matrix derived in (i). (6)

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