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**B.E.,/B.Tech.,(Full time) Degree End Semester Examinations, May 2013**

COMPUTER SCIENCE ENGINEERING & INFORMATION TECHNOLOGY

MA9265 - DISCRETE MATHEMATICS

FOURTH SEMESTER

(REGULATION 2008)

Time: Three Hours | Max.:100 Marks

Answer ALL Questions

PART-A (10 x 2 = 20 Marks)

1. Write the truth table for $(p \vee \neg q) \rightarrow (p \wedge q)$.

2. Prove that " if $n$ is an integer and $n^3+5$ is odd, then $n$ is even" using a proof by contradiction.

3. How many non-negative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$ ?

4. A drawer contains a dozen brown socks and a dozen black socks. A man takes socks at random in dark. What is the minimum number of socks he must take out to be sure that he has atleast two socks of the same color?

5. For what values of $m, n$, the complete bipartite graph $K_{m,n}$ is Eulerian?

6. Show that in any simple graph, the number of veritces of odd degree is even.

7. Is every Abelian group cyclic? Justify your answer.

8. Define an integral domain and give an example.

9. Let $S = \{a, b, c\}$. Draw the Hasse diagram of $\langle P(S), \subseteq \rangle$ where $P(S)$ is the power set of $S$.

10. Give an example of a lattice which is complemented but not distributive. Justify your answer.

PART - B (5 × 16 = 80 Marks)

11. i. Solve the recurrence relation $a_{n+2} = a_{n+1} + a_n, n \geq 0$ with $a_0 = 0, a_1 = 1$. (6)

ii. Find the number of integers between 1 and 500 (both inclusive) that are not divisible by any of the integers 2, 3, 5 and 7. (10)

12. (a) i. Obtain the principal disjunctive and conjunctive normal forms of the formula $(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$ (8)

ii. Prove the implication
$\forall x(P(x) \to Q(x)), \forall x(R(x) \to \neg Q(x)) \Rightarrow \forall x(R(x) \to \neg P(x))$ (8)

(or)

(b) i. Construct an argument to show that the following premises imply the conclusion " it rained ".
" If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on "; " If the sports day is held, the trophy will be awarded " and " the trophy was not awarded ". (10)

ii. Show that $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$ (6)

13. (a) i. Prove that a connected graph $G$ is bipartite if and only if all its cycles are of even length. (10)

ii. Define complement of a graph and prove that a self complementary graph has $4n$ or $4n+1$ vertices where $n$ is any positive integer. (6)

(or)

(b) i. Draw the graphs whose adjacency matrices are given below and check whether they are isomorphic or not.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$ (8)

ii. If $G$ is a graph with $p$ vertices $(p \geq 3)$ and $\delta \geq p/2$, then prove that $G$ is Hamiltonian. (8)

14. (a) i. Prove that the order of a subgroup of a finite group is a divisor of the order of the group. (12)

ii. Prove that if $f : (G, *) \to (G', \Delta)$ is a homomorphism with kernal $K$, then $K$ is a normal subgroup of $G$. (4)

(or)

(b) i. Show that a nonempty subset $H$ of a group $(G, *)$ is a subgroup of $G$ if and only if $a, b \in H \Rightarrow a * b^{-1} \in H$. (8)

ii. If $G$ is a finite cyclic group of order $n$ generated by $a \in G$, then prove that $n$ is the least positive integer such that $a^n = e$ and $G = \{a, a^2, a^3 \ldots a^n\}$. (8)

15. (a) i. Let $(L, \leq)$ be a lattice. Prove that for any $a, b, c \in L$, the following inequalities hold.

$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$

$a * (b \oplus c) \geq (a * b) \oplus (a * c)$

where $a * b = glb(a, b)$ and $a \oplus b = lub(a, b)$ (8)

ii. Show that lub and glb of a subset of a poset is unique if it exists. (4)

iii. Prove that cancellation laws hold good in a distributive lattice. (4)

(or)

(b) i. Let $(L, \leq)$ be a lattice and $a * b = glb(a, b)$ and $a \oplus b = lub(a, b)$. Prove that $*$ and $\oplus$ satisfy the following properties.

1. Idempotent 2. Associative 3. Commutative 4. Absorption (8)

ii. Show that the following is true in a complemented and distributive lattice. (8)

$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$ where $a'$ denotes the complement of $a$.