Regn. No: | | | | | | | | | | |

Anna University, Chennai - 600 025
B.E / B.Tech (Full Time) Degree End Semester Examinations - Septmber 2013
II Semester B.E /B.Tech - Common to All Branches
MA132 / MA181 / MA9161 - Mathematics II- (Regulations 2008)

Duration: 3 Hours — Total marks= 100

Part A — (10 x 2 = 20 Marks)

1. Find the particular integral of $(D^2 - 2D + 1)y = x^2$. x^2+4x+6
2. Solve $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$. $y = Ax + Bx\log x$
3. Find $\bar{\nabla} \circ \bar{\nabla}\phi$ for the function $\phi(x, y, z)$. $\nabla^2\phi$
4. If C is a planar curve connecting two points A and B, find $\int_C \bar{\nabla}\phi \cdot d\bar{r}$. $\phi_B - \phi_A$
5. If $f(z)$ is analytic and $\overline{f(z)}$ also analytic, then show that $f(z)$ is constant. $f = u+iv$ $u = v = \text{const}$
6. Find the image of $|z - 2| = 1$ under the mapping $w = z - 2 + 5i$. $|w-5i|=1$ $C(0,5i)$ $r=1$.
7. Evaluate $\int_C (505z^2 + 600z + 1000)dz$, where C is the circle $|z| = 101$. $= 0$
8. Identify and classify the singularity of the function $f(z) = \sin z/z$. removable sing.
9. Write down the Laplace transform of a periodic function $f(t)$ with period "a". $\frac{1}{1-e^{-as}}\int_0^a e^{-st}f(t)dt$
10. Using Initial Value theorem, find $\lim_{t\to 0} f(t)$, given that $L\{f(t)\} = \dfrac{1}{s(s+2)}$. $= 0$

Part B — (5 X 16=80 Marks)

11.a(i) Using method of variation of parameters solve $y'' - 2y' + y = xe^x$. $Ae^x + Bxe^x + \frac{x^3e^x}{6}$ (8 Marks)

a(ii) Using method of undetermined coefficients solve $y'' + y = 10\sin x$. (8 Marks)
$A\cos x + B\sin x - 5x\cos x$.

12.a(i) Find $\oint_C \left[(x^2 - y^2)dx + 2xydy\right]$, where C is the boundary of the square in the $xy-$ plane formed by $x = 0, y = 0, x = 1$ and $y = 1$. $= 2\left(\frac{1}{2} + 1 + \frac{2}{3} + 0\right)$ (8 Marks)

(ii) Using Stoke's theorem find $\int_C \bar{F}\phi \cdot d\bar{r}$ for the vector field $\overline{\mathrm{F}} = -y\hat{\mathrm{i}} + 2yz\hat{\mathrm{j}} + y^2\hat{\mathrm{k}}$ where S is hemisphere $x^2 + y^2 + z^2 = 1$ and C is its boundary on the $xy-$ plane. π (8 Marks)

(OR)

12.b(i) Verify Gauss divergence theorem for the vector function $\bar{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. 3 (10 Marks)

(ii) Find $\phi(x, y, z)$ with $\phi(1, -2, 2) = 4$ if $\bar{\nabla}\phi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$. (6 Marks)
$\phi = x^2yz^3 + 20$

13.a(i) Find the analytic function $f(z) = u(x,y) + iv(x,y)$, given that $u(x,y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. Further, find $v(x,y)$. $= 3x^2y - y^3 + 2xy$ (8 Marks)

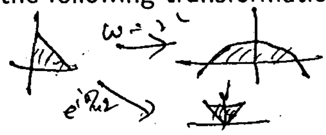
(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. Find also the pre-image of $|w| = 1$ under this bilinear transformation. $w = i\left(\frac{z-1}{z+1}\right)$ $x=0$ (8 Marks)

(OR)

13.b Find the image (in w-plane) of the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ in z-plane under the following transformations:

(i) $w = z^2$ (8 Marks)

(ii) $w = e^{i\pi/4}z$ (8 Marks)



$v = \frac{1-u^2}{2}$

14.a(i) Evaluate $\int_C \frac{dz}{z^2(z+2)}$ where C is the circle in Z-plane: $|z-1| = 2$ $-\frac{\pi i}{2}$ (8 Marks)

(ii) Find the Laurent's series expansion of $f(z) = \frac{7z-2}{(z+1)z(2-z)}$ valid in the region $1 < |z+1| < 3$. (8 Marks)

(OR)

14.b(i) Find the Taylor power series expansion of $f(z) = \sin z$ about the point $z = \pi/4$. (8 Marks)

(ii) Using Contour integration on unit circle, evaluate $\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)}$. $= \frac{2\pi}{\sqrt{3}}$ (8 Marks)

15.a(i) Evaluate $\int_0^{\infty} te^{-3t}\sin t\,dt$. $\frac{2s}{(s^2+1)^2}$ $\frac{6}{100}$ (4 Marks)

(ii) Find the inverse Laplace transform $L^{-1}\left(\frac{s}{s^2+2s+2}\right)$. $e^{-t}(\cos t - \sin t)$ (6 Marks)

(ii) Find the Laplace transform $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$. $\log\left(\frac{s+b}{s+a}\right)$ (6 Marks)

(OR)

$t\sin t$

15.b(i) Using convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+1)}$. (8 Marks)

(ii) Solve $y'' - 6y' + 9y = t^2e^{3t}, y(0) = 2, y'(0) = 6$ by Laplace transform method,. (8 Marks)

–Paper Ends– $y(t) = 2e^{3t} + \frac{1}{20}t^5e^{3t}$