Reg. No. | | | | | | | | | | |

B.E./ B. Tech (Full Time) DEGREE END SEMESTER EXAMINATION, Nov / Dec - 2013

THIRD SEMESTER

MA 9211/ MA231/ MA271 - MATHEMATICS - III (Regulation – 2002/ 2004/ 2008)

COMMON TO ALL BRANCHES

Time: 3 hours **Maximum: 100 Marks**

Answer ALL Questions

Part – A $(10\times 2 = 20$ marks)

1. State Dirichlet's conditions for existence of Fourier series.
2. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$, in $0 < x < 2\pi$, then deduce the value of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$.
3. If the Fourier transform of $f(x)$ is $F(s)$, then find the Fourier transform $e^{iax}f(x)$.
4. Solve for $f(x)$ if $\int_0^{\infty} f(x)\cos\alpha x\, dx = e^{-\alpha}$.
5. Find the complete solution of $xy = pq$.
6. Find the general solution of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = 0$.
7. What are the possible solutions of the one dimensional heat flow equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.
8. What are the assumptions made in deriving the one dimensional wave equation?
9. Find the Z-transform of na^n.
10. If $Z\{f(n)\} = \frac{z^2}{z^2+1}$, then find $\lim_{n\to\infty} f(n)$.

Part – B $(5\times 16 = 80$ marks)

11. i) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & if\ |x| < 1 \\ 0, & otherwise \end{cases}$.

    Hence deduce that $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. (8)

    ii) Find the Fourier sine transform of e^{-ax} $(a > 0)$, hence deduce that $\int_0^{\infty}\frac{x^2 dx}{(x^2+a^2)^2} = \frac{\pi}{4a}$. (8)

12. (a) i) Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & if\ -\pi < x < 0 \\ x, & if\ \ 0 < x < \pi \end{cases}$. (8)

ii) Obtain the Fourier cosine series expansion of $f(x) = x$ in $0 < x < 4$. Hence deduce the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...to\ \infty$. (8)

(OR)

(b) i) Find the Fourier series expansion of $f(x) = x^2$ in $-\pi < x < \pi$. Hence deduce for $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (8)

ii) Compute the first two harmonics of the Fourier series of $f(x)$ from the table: (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

13. (a) i) Find the partial differential equation by eliminating the arbitrary functions f and g from $z(x,y) = xf(y) + yg(x)$. (8)

ii) Find the general solution of $z(x-y) = px^2 - qy^2$. (8)

(OR)

(b) i) Find the singular integral of $z = px + qy + p^2q^2$. (8)

ii) Find the general solution of $(D^2 - 2DD' + D'^2)z = \cos(x-3y) + xy^2$. (8)

14. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$, find the displacement of the string in the subsequent time. (16)

(OR)

(b) An infinitely long plane uniform plate is bounded by two parallel edges $x = 0$ and $x = l$, and an end at right angles to them. The breadth of this edge $y = 0$ is l and is maintained at a temperature 100° and all the other three edges are at temperature zero. Find the steady state temperature at any interior point of the plate. (16)

15. (a) i) Find the inverse Z-transform of $\frac{z^3 - 20z}{(z-4)(z-2)^3}$. (8)

ii) Find the Z-transform of $\frac{1}{n(n+1)}$, for $n \geq 1$. (8)

(OR)

(b) i) Using convolutions, find the inverse Z-transform of $\frac{z^3}{(z-a)^3}$. (8)

ii) Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = 0, u_1 = 0$, by using Z-transforms. (8)

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