Reg. No.:

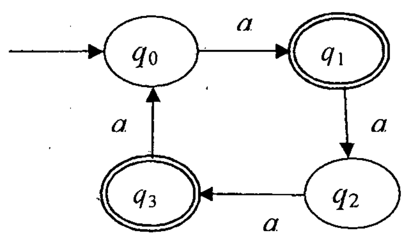
B.E. DEGREE EXAMINATIONS NOV/DEC 2012
FIFTH SEMESTER
CS 9302 – THEORY OF COMPUTATION
REGULATIONS – 2008

Time : 3 hours **Max. Marks : 100**

<u>Answer ALL Questions</u>

<u>PART A – (10 x 2 = 20 marks)</u>

1. Prove by contrapositive method that if the product of two integers is odd, then both the integers must be odd.
2. Give a DFA that accepts odd number of 0s over $\Sigma = \{0,1\}$.
3. If L, M and N are regular languages, prove that $L(M \cup N) = LM \cup LN$.
4. Construct a minimum state DFA equivalent to the following DFA using table filling algorithm:



5. Show that the following context free grammar is ambiguous.
   $S \rightarrow a|Sa|bSS|SSb|SbS$
6. Design a context free grammar for the language $L = \{a^n b^m \mid n, m \geq 1, n \neq m\}$
7. Show that context free languages are closed under union.
8. Convert the following grammar to Chomsky Normal form.
   $S \rightarrow bA \mid aB$
   $B \rightarrow aBB \mid b \mid bS$
   $A \rightarrow a \mid aS \mid bAA$
9. Prove that the complement of a recursive language is recursive.
10. Give the binary code for the Turing machine $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, q_4)$ where δ is given as

$\delta(q_0,0) = (q_1, 0, R)$
$\delta(q_1,0) = (q_2, 0, R)$
$\delta(q_2,0) = (q_3, 1, R)$
$\delta(q_2,1) = (q_3, 1, L)$
$\delta(q_2,B) = (q_3, 1, R)$
$\delta(q_3,0) = (q_2, 1, L)$
$\delta(q_3,1) = (q_4, 1, R)$
$\delta(q_3,B) = (q_2, 1, L)$

PART B – (5 x 16 = 80 marks)

11. (i) If $D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ is the DFA constructed from NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ by the subset construction, then prove that $L(D) = L(N)$. (7)

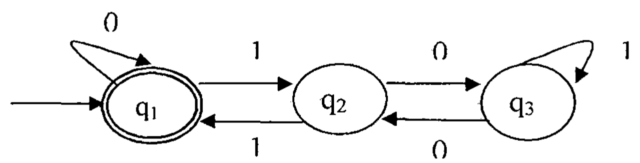
(ii) Construct a DFA $D1$ that will accept the language $L1$ where $L1$ is the set of all strings with 110 as substring. Construct another DFA $D2$ that will accept the language $L2$ where $L2$ is the set of all strings with 10 as substring. Construct DFAs $D3$, $D4$ and $D5$ that will accept the languages $L1 \cap L2$, the complement of $L1$ and $L1 \cup L2$ respectively. (9)

12. (a) (i) Write a regular expression that will accept strings with zero or more a's followed by zero or more b's followed by zero or more c's. Draw the equivalent ε-NFA. Convert the ε-NFA into a DFA. (10)

(ii) Prove using pumping lemma that the language $L = \{a^i b^j \mid i \leq 2j\}$ is not regular. (6)

(OR)

12. (b) (i) Find the regular expression for the following DFA using Kleene's algorithm. (10)



(ii) Prove or disprove the following statement about regular expressions:

$(RS+R)^*RS = (RR^*S)^*$

where R and S are regular expressions. (6)

13. (a) (i) Let $G = (V, T, P, S)$ be a context free grammar, and suppose there is a derivation $A \overset{*}{\underset{G}{\Rightarrow}} w$, where w is in T^*. Prove that the recursive inference procedure applied to G determines that w is in the language of variable A. (8)

(ii) Design a push down automaton that will accept the set of all strings of 0s and 1s such that the number of 1s is twice the number of 0s. (8)

(OR)

13. (b) (i) If $L = N(P_N)$ for some PDA $P_N = (Q, \Sigma, \Gamma, \delta_N, q_0, Z_0)$ that accepts by empty stack, then prove that there is a PDA P_F that accepts by final state such that $L = L(P_F)$. (8)

(ii) Convert the following push down automaton to a context free grammar. $P = (\{q_0, q_1, q_2, q_3\}, \{a,b\}, \{A, Z\}, \delta, q_0, Z, \emptyset)$, where δ is given by (8)

$\delta(q_0, a, Z) = \{(q_0, AZ)\}$

$\delta(q_0, b, A) = \{(q_1, \varepsilon)\}$

$\delta(q_0, a, A) = \{(q_3, \varepsilon)\}$

$\delta(q_1, \varepsilon, Z) = \{(q_2, \varepsilon)\}$

$\delta(q_3, \varepsilon, Z) = \{(q_0, AZ)\}$

14. (a) (i) If a context free grammar G_1 is constructed after eliminating ε- productions from the context free grammar G, prove that $L(G_1) = L(G) - \varepsilon$. (8)

(ii) Convert the following grammar to Greibach normal form.

$S \rightarrow XA \mid BB$
$B \rightarrow b \mid SB$
$X \rightarrow b$
$A \rightarrow a$ (8)

(OR)

14. (b) (i) Design a Turing machine that will accept strings containing equal number of 0s and 1s. (8)

(ii) Design a Turing machine that will take a binary number as input and will increment the number by one. (8)

15. (a) (i) Define L_d, the diagonal language. Prove that L_d is not recursively enumerable. (6)

(ii) Prove that L_u, the universal language is recursively enumerable but not recursive. (10)

(OR)

15. b) Consider the following Turing machine M = ($\{q_0, q_1, q_2, q_3, q_4\}$, {0, 1}, {0, 1, B}, δ, q_0, B, q_4) where δ is given as

$\delta(q_0,0) = (q_2, B, R)$
$\delta(q_0,1) = (q_1, B, R)$
$\delta(q_0,X) = (q_0, B, R)$
$\delta(q_0,B) = (q_4, B, R)$
$\delta(q_1,0) = (q_3, 0, L)$
$\delta(q_1,1) = (q_1, 1, R)$
$\delta(q_1,X) = (q_2, X, R)$
$\delta(q_2,0) = (q_2, 0, R)$
$\delta(q_2,1) = (q_3, X, L)$
$\delta(q_2,X) = (q_2, X, R)$
$\delta(q_3,0) = (q_3, 0, L)$
$\delta(q_3,1) = (q_3, 1, L)$
$\delta(q_3,X) = (q_3, X, L)$
$\delta(q_3,B) = (q_0, B, R)$

and input string w = 0101.

Construct a Modified Post's Correspondence Problem (MPCP) instance from the Turing machine M. Check if the MPCP instance has a solution. (16)