

ROLL NO.

B.E./B.Tech. (FULL TIME) DEGREE END SEMESTER EXAMINATIONS, APRIL/MAY 2011

GEO INFORMATICS BRANCH  
(COMMON TO MECHANICAL, MANUFACTURING, INDUSTRIAL, PRINTING AND MINING)

FOURTH SEMESTER

MA 503 – PROBABILITY AND STATISTICS

(REGULATIONS 2004)

Time : 3 Hours

Max. Marks : 100

**Instructions : Statistical Tables may be provided**

**Answer ALL Questions**

**Part – A ( 10 x 2 = 20 Marks)**

1. If A and B are independent events, then show that A and  $\bar{B}$  are also independent.
2. The odds against manager X settling the wage dispute with the workers are 8 : 6 and odds in favour of manager Y settling the same dispute are 14 : 16. Find the probability that neither settles the dispute, if they both try independently of each other.
3. Suppose the moment generating function of a random variable X is of the form  $M_X(t) = (0.4e^t + 0.6)^8$ . What is the moment generating function of  $Y=3X+2$ .
4. If X is a random variable and a and b are constants, then show that  $E(aX+b)=a E(X)+b$ , provided all the expectations exist for a continuous random variable .
5. State any two properties of regression coefficients.
6. If each of the two variables X and Y takes two values 0 and 1, with positive probabilities then  $r(X,Y) = 0$ , show that X and Y are independent.
7. Define type I and type II errors.
8. State any two applications of  $\chi^2$  – test.
9. What are the basic principles of an experimental design.
10. Define a randomized block design and write its analysis of variance table.

**Part – B ( 5 x 16 = 80 Marks)**

11. (a). (i). In 2011 there will be three candidates for the position of principal Mr. X, Mr. Y and Mr. Z whose chances of getting the appointment are in the proportion 4:2:3 respectively. The probability that Mr. X if selected would introduce co-education in the college is 0.3. The probabilities of Mr. Y and Mr. Z doing the same are respectively 0.5 and 0.8. What is the probability that there will be co-education in the college in 2011 and if there is a co-education in the college in 2011, what is the probability that Mr. Z is the principal. (8 Marks)

(ii). The diameter of a electric cable, say  $X$ , is assumed to be a continuous random variable with p.d.f.  $f(x) = 6x(1-x), 0 < x < 1$ . Check that  $f(x)$  is p.d.f., and determine a number 'b' such that  $P(X < b) = P(X > b)$ . (8 Marks)

12. (a). (i). If  $X$  and  $Y$  are independent poisson variates, find the conditional distribution of  $X$  given  $X+Y$ . (8 Marks)

(ii). Prove the memoryless property of an exponential distribution. (8 Marks)

**(OR)**

(b). (i). The mean yield for one-acre plot is 662 kilos with a standard deviation 32 kilos. Assuming normal distribution, how many one-acres plots in a batch of 1000 plots would you expect to have yield over 700 kilos and below 650 kilos. (8 Marks)

(ii). If  $X$  and  $Y$  are independent uniform variates on  $[0, 1]$ , find the distributions of  $XY$  and  $\frac{X}{Y}$ . (8 Marks)

13. (a). (i). The joint probability density function of a two-dimensional random variable  $(X, Y)$  is given by

$$f(x, y) = \begin{cases} 2; & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the marginal density functions of  $X$  and  $Y$ . Also find the conditional density function of  $Y$  given  $X = x$  and the conditional density function of  $X$  given  $Y=y$ . (8 Marks)

(ii). In the random placement of three balls in three cells, describe the possible outcomes of the experiment. Let  $X_i$  denote the number of balls in cell  $i$ ;  $i=1,2,3$ ; and  $N$ , the number of cells occupied. Obtain the joint distribution of  $(X_1, N)$ . (8 Marks)

**(OR)**

- (b). Calculate the correlation coefficient and the two lines of regression equations for the following data. Also obtain the estimate of X for Y=70.

X : 65 66 67 67 68 69 70 72

Y : 67 68 65 68 72 72 69 71.

(16 Marks)

14. (a). (i). The heights of six randomly chosen sailors are :63, 65, 68,69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71 72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers. Use 5% level of significance. (8 Marks)

- (ii). The following figures show the distribution of digits in numbers chosen at random from a telephone directory: Use 5% level of significance.

|            |      |      |     |     |      |     |      |     |     |     |
|------------|------|------|-----|-----|------|-----|------|-----|-----|-----|
| Digits :   | 0    | 1    | 2   | 3   | 4    | 5   | 6    | 7   | 8   | 9   |
| Frequency: | 1026 | 1107 | 997 | 966 | 1075 | 933 | 1107 | 972 | 964 | 853 |

Test whether the digits may be taken to occur equally frequently in the directory. (8 Marks)

(OR)

- (b). (i). Design a decision rule to test the hypothesis that a coin is fair if a sample of 64 tosses of the coin is taken and if a level of significance of 0.05 and 0.01 is used. (8 Marks)

- (ii). The manufacturer of a patent medicine claimed that it was 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy the medicine provided relief for 160 people. Determine whether the manufacturer's claim is legitimate. Use 1% level of significance. (8 Marks)

15. (a). Table below shows the yields per hectare of four different plant crops grown on lots treated with three different types of fertilizers. Test at 0.01 level of significance (i). whether there is a significant difference in yield per hectare due to fertilizers and (ii). there is a significant difference in yield per hectare due to crops.

|              | Crop I | Crop II | Crop III | Crop IV |            |
|--------------|--------|---------|----------|---------|------------|
| Fertilizer A | 4.5    | 6.4     | 7.2      | 6.7     |            |
| Fertilizer B | 8.8    | 7.8     | 9.6      | 7.0     |            |
| Fertilizer C | 5.9    | 6.8     | 5.7      | 5.2     | (16 Marks) |

(OR)

- (b). Explain briefly the advantages and disadvantages of randomized block design and latin square design. (16 Marks).

\$\$\$\$