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B.E/B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, APRIL / MAY 2011

ELECTRONICS AND COMMUNICATION ENGINEERING

FOURTH SEMESTER **(REGULATION 2004)**

MA504 – RANDOM PROCESSES

Time : 3 hrs Max. Marks : 100

Answer ALL Questions

Part – A (10 x 2 = 20 Marks)

1. If $P(A)=0.5,\ P(B)=0.3$ and $P(A\cap B)=0.15$, find $P\left(A/B\right)$ and $P(\bar{A}\cap\bar{B})$.

2. If the p.d.f of a random variable X is $f(x)=\begin{cases}2x, & 0\le x\le 1\\ 0, & otherwise\end{cases}$, find $E(2X+3)$ and $P\left(X<\frac{1}{2}\right)$.

3. Let $P(X=2)=\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x=1,2,3\ldots$, be a probability mass function of a random variable X. Find $P(X\ge 4)$.

4. The random variable X has the p.d.f. $f(x)=\frac{1}{5}e^{-\frac{1}{5}x}, x>0$. Find the moment generating function (M.G.F.) of X and hence obtain $E(X)$.

5. The joint p.d.f. of the random variables X and Y is given by $f(x,y)=\frac{3}{8}xy(2-x),\quad 0\le x\le 2,\ 0\le y\le 2$. Find $P(X>Y)$.

6. Show that the correlation coefficient ρ_{XY} of the random variables X and Y lies between -1 and 1.

7. Define strict sense stationary and wide sense stationary for random processes.

8. Let $\{X(t);t\ge 0\}$ be a Poisson process with arrival rate λ. Write expressions for $P(X(t)=n)$ and $E(X(t))$.

9. For two jointly wide sense stationary random processes, the cross-correlation function is $R_{XY}(\tau)=\begin{cases}2e^{-2\tau}, & \tau>0\\ 0, & \tau\le 0\end{cases}$. Find the cross-spectral densities $S_{XY}(\omega)$ and $S_{YX}(\omega)$.

10. Define white noise random process.

PART – B (5 x 16 = 80 Marks)

11. (i) If $X(t)$ and $Y(t)$ are two wide sense stationary random process, prove that

$$|R_{XY}(\tau)| \le \frac{1}{2}\left[R_{XX}(0) + R_{YY}(0)\right].$$

(ii) The power spectral density of a wide sense stationary process is $S_{XX}(\omega) = \frac{1}{\left(1+\omega^2\right)^2}$. Find the auto-correlation function and the average power of the process $\{X(t)\}$.

(iii) A random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \ge 0$. If the auto-correlation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $Y(t)$.

12. a.(i) A consulting firm rents cars from three agencies in the following manner: 20% from agency D, 20% from agency E and 60% from agency F. If 10% of the cars from D, 12% of the cars from E and 4% of the cars from F have bad tyres, what is the probability that the firm will get a car with bad tyres? Find the probability that a car with bad tyres is rented from agency three?

(ii) Suppose that the random variable X has the cumulative distribution function (C.D.F.) $F(x) = 1-(1+x)e^{-x}, x \ge 0$.

Find (1) $P(1 < X \le 2)$ (2) $P(X \ge 2)$ (3) $E(3X+2)$ and (4) $Var(3X+2)$

(OR)

b.(i) A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good, what is the probability that other one is also good?

(ii) If A and B are independent events, show that $\overline{A}$ and $\overline{B}$ are also independent events.

(iii) The p.d.f. of a random variable X is given by $f(x) = \begin{cases} kx(2-x), & 0 < x < 2 \\ 0, & otherwise \end{cases}$.

Find (1) the value of k (2) cumulative distribution of X (3) $P\left(X > \frac{1}{2}\right)$.

13. a.(i) Suppose the random variable X has a geometric distribution with p = 0.5.

Determine (1) $P(X \le 2)$ (2) $P(X > 2)$ (3) $Var(X)$

(ii) Determine the binomial distribution for which $E(X)=4$ and $Var(X)=3$. Obtain its moment generating function also.

(iii) Find the probability density function of the random variable $Y = X^2$ where X is the standard normal random variable.

(OR)

b.(i) If a random variable X has the p.d.f. $f(x)=\begin{cases} xe^{-x}, & x>0 \\ 0, & otherwise \end{cases}$, find

(1) M.G.F. of X and obtain $E(X)$ and $Var(X)$ (2) $P(X>2)$ (3) C.D.F. of X.

(ii) The moment generating function $M_X(t)$ of a random variable X is given as

$$M_X(t)=\frac{e^{15t}-e^{3t}}{12t},\ t\neq 0.$$

Find (1) the distribution of X (2) $E(X)$ (3) $Var(X)$ (4) $P(4<X<8)$.

14. a.(i) If the joint p.d.f. of random variables (X,Y) is given as

$$f(x,y)=\begin{cases} e^{-(x+y)}, & x\geq 0,\ y\geq 0 \\ 0, & otherwise \end{cases}.$$

Find (1) $P(X>2, Y>4)$ (2) $P(X>Y)$ (3) $P(X+Y>1)$ (4) $E(XY)$.

(ii) Find the correlation coefficient between random variables X and Y if the joint p.d.f. of X and Y is $f(x,y)=\begin{cases} 2, & x>0,\ y>0,\ x+y<1 \\ 0, & otherwise \end{cases}$.

(OR)

b.(i) The joint p.d.f. of the random variable (X,Y) is given by

$$f(x,y)=\begin{cases} \dfrac{x\left(1+3y^2\right)}{4}, & 0<x<2,\ 0<y<1 \\ 0 & otherwise \end{cases}.$$

Find (1) marginal p.d.f.s of X and Y (2) the conditional p.d.f. of X given Y

(3) $P\left(\frac{1}{4}<X<\frac{1}{2} \,/\, Y=\frac{1}{3}\right)$ (4) Are X and Y independent R.Vs? Explain.

(ii) Let X and Y be two independent R.Vs with joint p.d.f. $f(x,y)=\begin{cases} e^{-(x+y)}, & x>0, y>0 \\ 0, & otherwise \end{cases}$.

Find the joint p.d.f. of $U=X+Y$ and $V=\frac{X}{Y}$. Are U and V independent R.Vs?

15. a.(i) A random process is given as $X(t) = A + B\cos(\omega t + \theta)$ where 'A' is a random variable that is uniformly distributed between -3 and $+3$, 'B' is a random variable with $E(B) = 0$ and $Var(B) = 4$, 'ω' is a constant and θ is a random variable that is uniformly distributed from $-\pi/2$ to $3\pi/2$. Here A, B and θ are independent R.Vs. Compute $E(X(t))$ and $E(X(t)X(t+\tau))$, $\tau > 0$. Is the process $X(t)$ stationary in the wide sense? Explain.

(ii) Consider a Markov chain $\{X_n; n = 0,1,2...\}$ having state space $S = \{1,2,3,4\}$ and one-step transition probability matrix $P = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$.

(1) Draw a transition diagram (2) Is the chain irreducible?
(3) Is the state-2 ergodic? Explain. (4) Is the state-3 transient? Explain.

(OR)

b.(i) Show that the interarrival time between two consequent arrivals of a Poisson process with parameter λ follows an exponential distribution.

(ii) Derive the auto-correlation function of a Poisson process $\{X(t); t \geq 0\}$ with rate λ.

(iii) Let $\{X_n;\ n = 0,1,2...\}$ be a Markov Chain having state space $S = \{0,1,2\}$ with one-step transition probability matrix $P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $P(X_0 = i) = 1/3$, $i = 0,1,2$.

Find (1) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 0)$ (2) $P(X_2 = 2, X_1 = 1/X_0 = 2)$
(3) $P\{X_2 = 1/X_0 = 0\}$ (4) $P\{X_2 = 1, X_0 = 0\}$
