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Registration number | | | | | | | | | |

B.E/B.Tech.(Full time) DEGREE

END SEMESTER ARREAR EXAMINATIONS, NOV/DEC 2012

COMPUTER SCIENCE AND ENGINEERING BRANCH

SIXTH SEMESTER

MA9266 - PROBABILITY AND QUEUEING THEORY

(REGULATIONS 2008)

(Use of Statistical Tables and Calculators are permitted)

Time: 3 Hours ANSWER ALL QUESTIONS Max Mark : 100

PART -A (10 x 2 = 20 Marks)

1. If the probability density of a random variable X is given by $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & otherwise \end{cases}$, determine the moment generating function of X.

2. Suppose that the number of typographical errors on a single page of the book has a Poisson distribution with parameter $\lambda = 1/2$. Calculate the probability that there is atleast one error on the given page.

3. The joint probability density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} c(1-x)(1-y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the constant c.

4. Let X and Y be any two random variables and a, b be constants. Prove that $Cov(aX, bY) = ab\ Cov(X, y)$.

5. Define strict senses and wide sense stationary process.

6. A random process has a autocorrelation function

$$R_{XX}(\tau) = \frac{16\tau^2 + 28}{\tau^2 + 1}$$

Determine the variance of the process.

7. An average of 10 cars/hour arrive at a drive in teller. Assume that the average service time for each customer is 4 minutes and both interarrival and service times are exponential. What is the probability that the teller is idle?

8. State Little's Formula for a $(M/M/1) : (N/FIFO)$ queueing model.

9. Distinguish between open and closed queueing networks.

10. Show that for a case when the service time is constant with mean $1/\mu$, the Pollaczek-Khintchine formula reduces to $L_S = \rho + \dfrac{\rho^2}{2(1-\rho)}$.

PART B (5 x 16 = 80 Marks)

11. (a) Derive the Pollaczek-Khintchine formula for the average number in the system in a $M/G/1$ queueing model. (10 Marks)

(b) For a open queueing network with three nodes 1,2 and 3, let customers arrive from outside the system to node j according to a Poisson input process with parameter r_j and let p_{ij} denote the proportion of customers departing from facility i who goes next to facility j. Given $(r_1, r_2, r_3) = (1, 4, 3)$ and the probability matrix P_{ij} as $\begin{pmatrix} 0 & 0.6 & 0.3 \\ 0.1 & 0 & 0.3 \\ 0.4 & 0.4 & 0 \end{pmatrix}$, determine the average arrival rate λ_j to the node j for $j = 1, 2, 3$.

(6 Marks)

12. (a) i. A random variable X has the density function given by $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & otherwise \end{cases}$, Find the first four moments about the origin.

(8 Marks)

ii. If X is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$, find the mean and variance of X. (8 Marks)

[OR]

(b) i. State and prove the memoryless property of an exponential random variable. (8 Marks)

ii. Let X be a random variable with probability density function $f_X(x)$. Let $Y = X^2$, find the probability density function of Y. (8 Marks)

13. (a) i. Let X and Y be continuous random variable with the joint density function

$$f(x,y) = \begin{cases} \frac{x}{5} + cy & 0 < x < 1,\ 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find c. Are X and Y independent? (8 Marks)

ii. If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. (8 Marks)

[OR]

(b) The probability density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the lines of regression of X on Y and Y on X. (16 Marks)

14. (a) i. Consider a random process $X(t)$ defined by $X(t) = Y\cos(\omega t + \Theta)$ where Y and Θ are independent random variables and are uniformly distributed over $(-A, A)$ and $(-\pi, \pi)$ respectively. Is $X(t)$ a WSS process?

(8 Marks)

ii. If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson process with parameter λ_1 and λ_2 respectively, show that

$$P(N_1(t) = k/N_1(t) + N_2(t) = n) = \binom{n}{k} p^k q^{n-k}$$

where $p = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \dfrac{\lambda_2}{\lambda_1 + \lambda_2}$.

(8 Marks)

[OR]

(b) i. A random process $X(t)$ is defined by $X(t) = Acost + Bsint, -\infty < t < \infty$, where A and B are independent random variables each of which has a value -2 with probability 1/3 and a value 1 with probability 2/3. Is X(t) a wide-sense stationary process?

(8 Marks)

ii. Let $\{X_n\}$ be a Markov chain with state space $\{0, 1, 2\}$ with initial probability vector $p^{(0)} = (0.7, 0.2, 0.1)$ and the one step transition probability matrix

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

Compute (A) $P(X_2 = 3)$ (B) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

(8 Marks)

15. (a) i. A gas station has one diesel fuel pump for trucks only and has room for three trucks(including one at the pump). On the average trucks arrive at the rate of 4 per hour, and each truck takes 10 minutes to service. Assume that the arrival process is Poisson and the service time is an exponential random variable. What is the average time of the truck from entering to leaving the station? What is the average time of the truck to wait for service? What percentage of the truck traffic has been turned away?

(8 Marks)

ii. Obtain the steady state probabilities for the $(M/M/1) : (FCFS/N/\infty)$ queueing model and hence derive an expression for the average number of customers in the queue. (8 Marks)

[OR]

(b) i. A small bank has two tellers who are equally efficient and who are each capable of handling an average of 60 customer transactions per hour, with the actual service times exponentially distributed. Customers arrive at a bank according to a Poisson process at a mean rate of 100 per hour. Determine the probability that there are more than three customers in the bank. Also find the probability that a given teller is idle. (8 Marks)

ii. Derive the steady state probabilities for the finite source queueing model.

(8 Marks)

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