10141

B.E / B.Tech (Full Time) Degree End Semester Examinations MAY/JUNE 2013

Sixth Semester – Regulation 2008

Common to Computer Science Engineering and Computer Technology

MA 9266 Probability and Queueing Theory

25

Time: 3 Hours Max: 100 Marks

Answer **ALL** Questions

Part – A (10×2 = 20 Marks)

1. Let X be a discrete R.V. with probability mass function $P(X = x) = \begin{cases} \frac{x}{10}, & x = 1,2,3,4 \\ 0, & otherwise. \end{cases}$

   Find $P(X < 3)$ and $E(\frac{1}{2}X)$.

2. If the p.d.f of the continuous R.V. X is given as $f(x) = e^{-x}$, x > 0, find the p.d.f of the R.V. $Y = 2X + 1$.

3. The joint p.d.f of random variables X and Y is given as $f(x,y) = \begin{cases} 10xy^2, & 0 \le x \le y \le 1 \\ 0, & otherwise. \end{cases}$

   Find the marginal p.d.f of the R.V. X.

4. State the central limit theorem for independent and identically distributed random variables.

5. Derive the autocorrelation function for a Poisson process with rate λ.

6. Let $\{X_n : n \ge 0\}$ be a Markov chain having state space S = {0, 1} and one-step TPM $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$.

   Draw the transition diagram and identify the absorbing, recurrent and transient states?

7. In a given $M/M/1/\infty$ FCFS queueing system, the average rate of arrival is 2 customers per minute; $\rho = 0.6$. Find the mean number of customers in the system and the mean waiting time in the system.

8. For M/M/C/N FCFS (C < N) queueing system, write expressions for P_0 and P_N.

9. An M/D/1 queue has an arrival rate of 10 customers per second and a service rate of 20 customers per second. Compute the mean number of customers in the system and in the queue.

10. A repair facility shared by a large number of machines has 2 sequential stations with respective service rates of 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1 per hour. Assume that the system behavior may be approximated by the two-stage series queue. Find i) the mean repair time including the waiting time ii) the probability that both the service stations are idle.

Part-B (5× 16 = 80 Marks)

11. (i) Let $P(X = x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, x = 1, 2, 3,... , be the probability mass function of a R.V. X.

Find 1) P(X > 4) 2) P(X > 4 | X > 2) 3) the moment generating function $M_X(t)$ of the R.V. X and hence obtain E(X) and Var(X).

(ii) Let X be a uniformly distributed R.V over [-5, 5]. Find 1) $P(X \leq 2)$ 2) P(|X|>2) 3) P(X < 4 |X >2) 4) Cumulative distribution function of X 5) E(X) and Var(X).

12. (a) (i) Let the joint probability mass function of the R.V. (X,Y) be given as

$$P(X = x, Y = y) = \begin{cases} \frac{x+y}{12}, & x = 1, 2, \; y = 1, 2 \\ 0, & otherwise. \end{cases}$$

Find 1) the marginal probability mass functions of X and Y 2) $P(X + Y \leq 3)$ 3) P(X >Y) 4) Are the R.Vs X and Y independent? Explain.

(ii) If the random variables X and Y have joint p.d.f $f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, y > 0 \\ 0, & otherwise, \end{cases}$

find the joint p.d.f of the random variables $U = \frac{1}{X}$ and $V = X + Y$. Are the R.Vs U and V independent? Explain.

(OR)

(b) (i) If the joint p.d.f of the R.V (X, Y) is given as $f(x, y) = \begin{cases} 8xy, & 0 < x \leq y \leq 1 \\ 0, & otherwise. \end{cases}$

Find 1) the marginal p.d.fs of the R.Vs X and Y 2) f(y|x), the conditional p.d.f of the R.V Y given X = x 3) $P(Y < \frac{1}{8} | X < \frac{1}{2})$ 4) E(XY)

(ii) Let the R.Vs X and Y have the joint probability mass function

P(X=x, Y=y) = $\frac{x+2y}{18}, x = 1, 2, \; y = 1, 2.$ Find the correlation coefficient ρ_{XY} of the R.Vs X and Y.

13. (a) (i) A random process is given as $X(t) = U + V\cos(\omega t + \theta)$ where U is a random variable that is uniformly distributed between -3 and + 3, V is a random variable with E(V) = 0 and Var(V) = 4, ω is a constant and θ is a R.V that is uniformly distributed from $\frac{-\pi}{2}$ to $\frac{3\pi}{2}$. It is further assumed that U, V and θ are independent random variables. Is the process X(t) stationary in the wide-sense? Explain.

(ii) Let $\{X_n; n \geq 0\}$ be a Markov chain having state space S = {1,2,3} and one-step TPM

$$P=\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

1) Draw a transition diagram for this chain. 2) Is the chain irreducible? Explain 3) Is the state-3 ergodic? Explain 4) Find the invariant probabilities of the Markov chain.

(OR)

(b) (i) Let X(t) and Y(t) be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Find P(X(t) +Y(t) = n), n = 0, 1, 2,... and P(X(t) - Y(t) = n), n = 0, ± 1, ± 2,... .

(ii) Let $\{X_n; n \geq 0\}$ be a Markov chain having state space S = {1,2,3} with one-step TPM

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$$

and initial distributions $P(X_0 = i) = \frac{1}{3}$, i = 1,2,3. 1) Is the state-2 ergodic? Explain.

2) Find $P\{X_3 = 2, X_2 = 1 \mid X_1 = 2, X_0 = 3\}$ 3) Find $P\{X_2 = 2, X_0 = 2\}$ and $P\{X_2 = 2 \mid X_0 = 1\}$.

14. (a) (i) A super market has two girls runnging up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if the customers arrive in a Poisson manner at the rate of 10 per hour. 1) What is the probability of an arriving customer has to wait for service? 2) What is the expected percentage of idle time for each girl? 3) What is the mean waiting time in the system and in the queue?

(ii) For an M/M/1/N FCFS queue, derive the system of differential difference equations for the system size probabilities. Obtain the corresponding steady-state equations and hence calculate the steady-state probabilities of the number of customers in the system and the mean number of customers in the system.

(OR)

(b) (i) Patients arrive at a clinic according to Poisson process at a rate of 30 patients per hour. The waiting room can not accommodate more than 14 patients. Examination time per patient is exponentially distributed random variable with rate of 20 per hour. 1) Find the effective arrival rate at the clinic 2) What is the probability that an arriving patient will not wait? 3) What is the expected waiting time until a patient is discharged from the clinic?

(ii) Derive the system of differential difference equations for the system size probabilities for a $M/M/C/\infty$ FCFS queueing system. Obtain the corresponding steady-state equations and hence calculate the steady- state probabilities of the number of customers in the system and the mean number of customers in the system.

15. (a) Discuss an $M/G/1/\infty$ FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula. Deduce also the mean system size for $M/M/1/\infty$ FCFS queue from the P-K formula.

(OR)

(b) Discuss the Jackson's open queuing network system and hence obtain 1) the product form solution of the system size probabilities 2) the mean number of customers in the system.