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ANNA UNIVERSITY

B.E/B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, NOV / DEC 2011

SIXTH SEMESTER (REGULATION 2008)

MA 9266 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science Engineering and Computer Technology)

Time : 3 hrs Max. Marks : 100

Answer ALL Questions

Part – A (10 x 2 = 20 Marks)

1. If the moment generating function (M.G.F.) $M_X(t)$ of the random variable X is $M_X(t) = e^{4(e^t-1)}$, find $P(X=6)$ and $E(X)$.

2. The random variable X has the p.d.f. $f(x) = 2e^{-2x}, x > 0$, find $P\left(X > \frac{1}{2}\right)$ and $E(X)$.

3. If the joint p.d.f. of the R.V.s X and Y is given as $f(x,y) = \begin{cases} \frac{1}{x}, & 0 < y < x \le 1 \\ 0, & otherwise \end{cases}$, find the marginal p.d.f. of the random variable Y.

4. Show that the correlation coefficient ρ_{XY} of the random variables X and Y lies between -1 and 1.

5. Define (i) wide-sense stationary random process and (ii) the n-th order stationary random process.

6. Let $\{X_n : n \ge 0\}$ be a Markov chain having state space $S = \{1,2\}$ and one-step TPM $P = \begin{bmatrix} 1 & 0 \\ 1/3 & 2/3 \end{bmatrix}$. Find the stationary / invariant probabilities of the Markov chain.

7. In an $M/M/1/\infty/FCFS$ queue, the arrival rate is $\lambda = 3$ customers / minute and utilization ratio is $\rho = 0.5$. Find L_s and W_S.

8. In an $M/M/C/N/FCFS$ queueing system, write expression for P_0 and P_N.

9. Consider a two-station tandem Markovian queueing network with service rates $\mu_1 = 4$ /minute at station–1 and $\mu_2 = 6$ /minute at station–2 and arrival rate $\lambda = 2$ /minute. Compute the mean number of customers in the system and mean delay a customer experiences in passing through the system.

10. State any two differences between Open and Closed Queueing Networks.

PART – B (5 x 16 = 80 Marks)

11. (i) Consider the R.V. $Y = \frac{1}{X}$ where the R.V. X is a uniformly distributed over (0,1). What is the p.d.f. of Y?

(ii) Suppose R.V. X is uniformly distributed over $(-a,a)$ and $a > 0$. (1) Find a if $P(X > 1) = \frac{1}{3}$ (2) Find the M.G.F. $M_X(t)$ of X and hence obtain $E(X)$ and $Var(X)$.

(iii) Determine the binomial distribution for which $E(X) = 4$ and $Var(X) = 3$. Find its MGF $M_X(t)$, $P(X = 2)$ and $Var(-3X + 4)$.

12. a.(i) Let X and Y be continuous R.Vs with joint p.d.f. $f(x,y) = \begin{cases} \frac{1}{3}x^3 e^{-x(1+y)}, & x > 0,\ y > 0 \\ 0\ , & otherwise \end{cases}$.

Find $Cov(X,Y)$.

(ii) If the joint p.d.f. of (X,Y) is given by $f(x,y) = \begin{cases} x + y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0\ , & otherwise \end{cases}$, find the j.p.d.f. of $U = XY$ and $V = Y$ and hence find the marginal p.d.f. of U.

(OR)

b.(i) Two random variables X and Y have joint p.d.f. $f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1 \\ 0\ , & otherwise \end{cases}$.

Find (1) the marginal p.d.fs of X and Y (2) the conditional p.d.f. of X given by $Y = y$ (3) $P\left(\frac{1}{4} < X < \frac{1}{2} / Y = \frac{1}{3}\right)$ (4) Are the random variables X and Y independent? Explain.

(ii) Let X and Y be two independent R.Vs with p.d.fs $f(x) = e^{-x}, x > 0$ and $g(y) = e^{-y}, y > 0$ respectively. Find the joint p.d.f. of $U = X + Y$ and $V = \frac{X}{Y}$. Are U and V independent R.Vs? Explain.

13. a.(i) A random process has sample function of the form $X(t) = A\cos(\omega t + \theta)$ where ω is a constant, A is a random variable that has a magnitude of $+1$ and -1 with equal probability and θ is a random variable that is uniformly distributed between 0 and 2π. Assume that the random variables A and θ are independent. Is $X(t)$ a wide-sense stationary process? Explain.

(ii) State the postulates of the Poisson process. Derive the system of differential and difference equations for Poisson process and hence obtain distribution and mean for that.

(OR)

b.(i) Let $\{X_n; n \geq 0\}$ be a Markov chain having state space $S = \{1,2\}$ with one-step TPM

$$P = \begin{bmatrix} \frac{6}{10} & \frac{4}{10} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$$ and the initial probability distribution $P(X_0 = i) = \frac{1}{2}$, $i = 1,2$.

(1) Is the chain irreducible? Explain.

(2) Find $P\{X_2 = 1, X_1 = 1 / X_0 = 2\}$, $P\{X_2 = 2,\ X_1 = 1,\ X_0 = 2\}$

(3) Is the state-1 ergodic? Explain.

(ii) Show that the interarrival time between two consecutive arrivals in a Poisson process is an exponential random variable. Is the Poisson process stationary? Explain.

14. a.(i) A petrol pump station has 4 pumps. The service time follows the exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at a rate of 30 cars per hour. (1) What is the probability that at arrival would have to wait in line? (2) Find the mean waiting time in the system and mean number of cars in the system. (3) What is the probability that the petrol pump station will be idle?

(ii) For an $M/M/1/N$ queue, derive the system of differential difference equations for the system size probabilities. Compute the steady – state probabilities of the system size and mean number of customers in the system.

(OR)

b.(i) Calculate the probabilities of the number of broken machines in the system in the steady state and also the mean number of broken machines in the system for the machine repair men problem which consists of K machines and $R(R < K)$ repair men.

(ii) Patients arrive at a clinic according to a Poisson process at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour. (1) Find the effective arrival rate at the clinic. (2) What is the probability that an arriving patient will not wait? (3) What is the expected waiting time until a patient is discharged from clinic?

15. a. Discuss an $M/G/1/\infty$ queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula. Deduce the mean number of customers for $M/M/1/\infty$ queue from the P-K mean value formula also.

(OR)

b. Discuss the open queueing network system and hence obtain (1) the product form solution of the system size probabilities (2) mean number of jobs in the system.
