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B.E. / B.Tech. (Full Time) DEGREE END SEMESTER EXAMINATIONS, NOV / DEC. 2011

ELECTRONICS AND COMMUNICATION ENGINEERING BRANCH

SEVENTH SEMESTER

EC9027 - INFORMATION THEORY

50

Time: 3 hr

(REGULATIONS 2008)

Max Mark: 100

Answer ALL Questions

Part – A (10 X 2 = 20 Marks)

1. Define Log sum inequality.
2. Let,  $X = \{0,1\}$  and consider two distributions  $p$  and  $q$  on  $X$ . Let,  $p(0) = 1-r$ ,  $p(1) = r$  and  $q(0) = 1-s$ ,  $q(1) = s$ . If,  $r = 1/2$  and  $s = 1/4$ . Find  $D(p||q)$  and  $D(q||p)$ .
3. State noiseless channel and find its capacity.
4. A high-resolution black and white TV picture consists of about  $2 \times 10^6$  picture elements and 16 different brightness levels. Pictures are repeated at the rate of 32 per second. All picture elements are assumed to be independent and all levels have equal likelihood of occurrence. Calculate the average rate of information conveyed by this TV picture source.
5. A channel is described by the following channel matrix, Draw the channel diagram and find the channel capacity.

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Verify,  $C_s = \log_2 m$ , where  $C_s$  is the channel capacity of a lossless channel and  $m$  is the number of symbols in  $X$ .
7. Find the Differential Entropy  $H(X)$  of the uniformly distributed random variable with probability density function.

$$f_x(x) = \begin{cases} 1/a, & 0 \leq x \leq a \\ 0, & \text{otherwise.} \end{cases}$$

8. Define Gaussian channel.
9. Write short notes on Broadcast channel.
10. State Relay channel.

Part – B ( 5 X 16 = 80 Marks)

11. (a). (i). Derive the expression for  $D(p(x,y)||q(x,y))$  (4)
- (ii). Prove,  $I(X,Y) = H(X) - H(X|Y)$  (4)
- (iii) Consider a BSC, with  $p(x_1) = \alpha$ , show that the mutual information  $I(X,Y)$  is given by,  $I(X,Y) = H(Y) + p \log_2 p + (1-p) \log_2 (1-p)$  and Calculate  $I(X,Y)$  for  $\alpha = 0.5$ ,  $p = 0.1$  (8)

12. (a).(i). An analog signal having 4 kHz bandwidth is sampled at 1.25 times the Nyquist rate, and each sample is quantized into one of 256 equally likely levels. Assume that the successive samples are statistically independent. What is the information rate of this source?, Can the output of this source be transmitted without error over an AWGN channel with a Bandwidth of 10KHz and S/N ratio of 20 dB? , Find Bandwidth required for an AWGN channel error-free transmission of the output of this source if the S/N ratio is 20 dB ?.

(10)

(ii). A binary channel has the following noise characteristic, if the input symbols are transmitted with probability  $\frac{3}{4}$  and  $\frac{1}{4}$ , find  $H(X/Y)$ ,  $H(Y/X)$ ,  $I(X,Y)$ .

(6)

(or)

(b).(i). Explain the Asymptotic Equipartition property in detail and derive its Codeword length.

(12)

(ii). A binary memoryless source X with two symbols  $x_1$  and  $x_2$ . Show that  $H(X)$  is maximum when both  $x_1$  and  $x_2$  are equiprobable.

(4)

13. (a).(i). Find the channel capacity of the binary Erasure channel.

(10)

(ii). Two random variables  $X \in \{0,1\}$  and  $Y \in \{0,1\}$ , with joint probability distribution  $P_{XY}(x,y)$  given by:  $P_{XY}(0,0)=1/2$ ,  $P_{XY}(0,1)=1/6$ ,  $P_{XY}(1,0)=1/6$ ,  $P_{XY}(1,1)=1/6$ . Calculate  $H(X,Y)$ ,  $H(X)$ ,  $H(Y)$ ,  $H(X/Y=0)$ ,  $H(X/Y=1)$ ,  $H(Y/X)$  and the Channel Capacity.

(6)

(or)

(b). (i).

$$P[a,b] = \begin{matrix} & b_1 & b_2 & b_3 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0.25 & 0 & 0.1 \\ 0 & 0.3 & 0.05 \\ 0.1 & 0.05 & 0 \\ 0 & 0 & 0.15 \end{bmatrix} \end{matrix}$$

Compute the transformation.

(10)

(ii). Consider a telegraph source having two symbols, dot and dash. The dot duration is 0.2 s. The dash duration is 3 times the dot duration. The probability of the dot's occurring is twice that of the dash, and the time between symbols is 0.2 s. Calculate the information rate of the telegraph source.

(6)

14. (a). Explain in detail about the Band limited channels.

(16)

(or)

(b).(i). Discuss the Parallel Gaussian channels in detail.

(8)

(ii). List the properties of Differential entropy.

(8)

15. (a). Explain in detail about the Gaussian multiple user channels with an Example.

(16)

(or)

(b). Discuss the Multiple Access channel in detail.

(16)