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Anna University, Chennai 600 025

B.E / B.Tech (Full Time) Degree End Semester Examinations - MAY 2012

II Semester B.E /B.Tech - Common to All Branches

MA9161 - Mathematics II- (Regulation 2008)

Duration: 3 Hours — Total marks= 100

Part A — (10 x 2 = 20 Marks)

1. Find the particular integral of $(D^2 - 2D)y = e^x \cos x$.
2. Guess the trial solution of the particular integral for the differential equation $y'' + y = \sin x$ using method of undetermined coeficients.
3. Find the directional derivative of $\phi = x^2y^2z^2$ at the point $(1, 1, 1)$ in the direction of the vector $2\hat{\mathrm{i}} + \hat{\mathrm{j}} + 2\hat{\mathrm{k}}$.
4. If $\overline{\mathrm{F}} = y^2\hat{\mathrm{i}} + 2(xy + z)\hat{\mathrm{j}} + 2y\hat{\mathrm{k}}$, state whether or not $\int_C \overline{\mathrm{F}} \cdot d\overline{\mathrm{r}}$ is path dependent.
5. If $f(z)$ is analytic and $\overline{f(z)}$ is also analytic, then show that $f(z)$ is constant.
6. Find the image of $|z - 2i| = 2$ under the mapping $w = z + 3 - 2i$.
7. Evaluate $\int_C \frac{5z^2 + 6z + 1000}{(z-1)^2} dz$, where C is the circle $|z| = 5/2$.
8. Identify and classify the singularity of the function $f(z) = (\cos z - 1)/z$.
9. Find the Laplace transform of $f(t) = t \sin t$.
10. Using Initial Value theorem, find $\lim_{t \to 0} f(t)$, given that $L\{f(t)\} = \frac{1}{s(s+2)}$.

Part B — (5 X 16=80 Marks)

11.a(i) Using method of variation of parameters solve the following differential equation $y'' - 2y' + y = xe^x$. (8 Marks)

11.a(i) Solve $(x^2D^2 - 3xD + 4)y = x \ \log x$. (8 Marks)

12.a(i) Verify Green's theorem for $\oint_C \left[(xy)dx + x^2dy\right]$, where C is the boundary of the square in the $xy-$ plane formed by $x = 0, y = 0, x = 1$ and $y = 1$. (8 Marks)

(ii) Verify Stoke's theorem for the vector field $\overline{\mathrm{F}} = -y\hat{\mathrm{i}} + 2yz\hat{\mathrm{j}} + y^2\hat{\mathrm{k}}$, where S is hemisphere $x^2 + y^2 + z^2 = 1$ and C is its boundary on the $xy-$ plane. (8 Marks)

(OR)

12.b(i) Verify Gauss divergence theorem for the vector function $\bar{F} = x\hat{i} + y\hat{j} + z\hat{k}$, where E is the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

(10 Marks)

(ii) Verify that $\overline{F} = y^2\hat{i} + 2xy\hat{j} + 2z\hat{k}$ is irrotational; further find also its corresponding scalar potential.

(6 Marks)

13.a(i) Find the analytic function $f(z) = u(x,y)+iv(x,y)$, given that $u(x,y) = 3x^2y+2x^2-y^3-2y^2$.

(8 Marks)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$ respectively. Find also the pre-image of $|w| = 1$ under this bilinear transformation. (8 Marks)

(OR)

13.b(i) Under the mapping $w = z^2$, find the image (in w-plane) of the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ in z-plane. Sketch vertices and boundary lines of original triangular object and the corresponding image object. (8 Marks)

(ii) Find the image of the circle $|z - i| = 1$ and the line $x + y = 1$ under the map $w = 1/z$.

(8 Marks)

14.a(i) Find the Laurent's series expansion of $f(z) = \dfrac{1}{z^2 - 3z + 2}$ valid in the region (i) $1 < |z| < 2$ (ii) $|z| > 2$.

(8 Marks)

(ii) Using Contour integration on unit circle, evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{(5 + 4\cos\theta)}$.

(8 Marks)

(OR)

14.b(i) Using Contour integration, evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{(x^2+9)(x^2+4)}dx$. (8 Marks)

(ii) Using Cauchy's residue theorem, evaluate $\displaystyle\int_C \frac{dz}{(z^2+4)^2}dz$, where $C : |z - i| = 2$. (8 Marks)

15.a(i) Solve $y'' - 2y' + y = te^t, y(0) = 1, y'(0) = 1$ by Laplace transform method. (8 Marks)

(ii) Using convolution theorem find the inverse Laplace transform of $\dfrac{s}{(s^2+1)^2}$. (8 Marks)

(OR)

15.b(i) Find the Laplace transform of the periodic function defined on the interval $0 \le t \le 1$ by

$$f(t) = \begin{cases} -1, & 0 \le t < 1/2 \\ 1, & 1/2 \le t < 1 \end{cases}$$ and $f(t+1) = f(t)$. (8 Marks)

(ii) Find the inverse Laplace transform of $\dfrac{s+1}{(s+1)(s^2+2s+2)}$. (8 Marks)

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