Anna University:Chennai-600 025
B.E/B.Tech Degree End Semester Examinations May 2012
Semester III-Common to All Branches-Regulation 2008
MA9211: MATHEMATICS III

17

Duration: 3 Hours | Answer All Questions | Total Marks: 100

Part A (10X2 = 20 Marks)

1. State by giving reasons whether the function $f(x) = \sin(\frac{1}{2x})$ satisfies the Dirichlet's conditions in the interval $-\pi \leq x \leq \pi$.
2. The Fourier series expansion of the function $f(x) = 2x^2$ in $-\pi \leq x \leq \pi$ is given by $\frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{8}{n^2}(-1)^n \cos(nx)$. Find the value of $\sum_{n=1}^{\infty} \frac{1}{n^4}$?
3. State Fourier Integral Theorem
4. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$
5. Solve $\frac{p^2 - q^2}{x - y} = 1$.
6. Form a partial differential equation by eliminating arbitrary functions from $z = f_1(x)f_2(y)$.
7. Write down all possible solutions of one dimensional heat flow equation
8. A bar 100 cm long, with insulated sides, has its ends kept at 0^oC and 100^oC until steady state conditions prevail. Find the temperature distribution.
9. From $y_n = a2^n + b(-2)^n$, derive a difference equation by eliminating the constants.
10. State final value theorem in Z-transforms

Part B:5x16=80 Marks

11 (i) Obtain the Fourier series for $f(x) = x - x^2$ in the interval $-\pi \leq x \leq \pi$. (10)

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 \leq x \leq 1$. (6)

12 a (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$

Hence evaluate $\int_0^\infty \frac{x\cos(x) - sin(x)}{x^3} \cos(\frac{x}{2}) dx$ (10)

(ii) Find the cosine transform of e^{-x} and hence evaluate $\int_0^\infty \frac{1}{(x^2+1)^2} dx$. (6)

OR

12 b (i) Find the cosine transform of $f(x) = \frac{1}{1+x^2}$ (10)

(ii) Using Fourier integral representation show that

$\int_0^\infty \frac{\omega \sin(x\omega)}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}$ for $x > 0$. (6)

13 a (i) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$ (8)

(ii) Solve $z^2(p^2 + q^2) = 2x + 3y$ (8)

OR

13 b (i) Solve $(mz - ny)p + (nx - lz)q = ly - mx$. (8)

(ii) Solve the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = e^{-x}$ (8)

14a A tightly stretched string is fastened to two points L apart. Motion is started by displacing the string in the form $y = 10x(L - x)$ from which it is released from rest at time $t = 0$. Find the displacement y(x,t) of any point at a distance x from one end at time t? (16)

OR

14b A rectangular plate with insulated surface is 8cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge y=0 is given by $U(x,0) = 100\sin(\pi x/8), 0 < x < 8$ while the two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at $o^0 C$, find the steady state temperature distribution of the plate. (16)

15 a (i) Using convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-2)(z-3)}\right]$ (6)

(ii) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms. (10)

OR

15 b (i) If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$ is the Z-transform of a sequence u_n, find u_2 and u_3 (8)

(ii) Find the Z-transform of $\cos(n\theta)$ and hence show that

$$Z[a^n \cos(n\theta)] = \frac{z(z - a\cos(\theta))}{Z^2 - 2az\cos(\theta) + a^2}$$

(8)