Registration Number | | | | | | | | | |

B.E./B.Tech (Full Time)DEGREE EXAMINATION - APRIL/MAY 2012
FOURTH SEMESTER-(REGULATION 2008)
(Common to Mechanical, Electrical and Electronics, Agriculture and Irrigation, Printing, Chemical, PRPC, Production, Automobile, Aeronautical and Rubber and Plastic)

MA 9262 NUMERICAL METHODS

Max Marks: 100 ANSWER ALL QUESTIONS Time: 3 Hours

PART A (10 × 2 = 20 Marks)

1. State the Newton-Raphson formula and the criteria for convergence.
2. What is total pivoting and partial pivoting?
3. Given a set of $n+1$ points, state the general form of the nth degree Lagrange interpolation polynomial.
4. Define a cubic spline.
5. Write the formula for the derivative $\frac{d^2y}{dx^2}$ at the point $x = x_0$ using Newton's forward difference interpolation formula.
6. What is three-point Gaussian quadrature formula? For what class of functions $f(x)$ does it give exact answers?
7. Illustrate Euler's method graphically.
8. What is the main drawback of Taylor's series method?
9. Is Crank-Nicholson method an implicit or explicit method? Justify.
10. When is the finite-difference procedure for solving a hyperbolic equation stable?

PART B (5 × 16 = 80 Marks)

11. (a) i. Using Newton's divided difference formula, find $f(8)$ given the following table:

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

(8)

ii. In the table given below, the values of y are consecutive terms of a series of which 23.6 is the 6th term. Find the first and tenth terms of the series:

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(8)

12. (a) i. Use Gauss-Jordan method to find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. (8)

ii. Find the largest eigen-value correct to three decimal places of the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ by power method, What is the corresponding eigen-vector? (8)

(Or)

(b) i. Solve the equations $83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$; $3x + 8y + 29z = 71$ by Gauss-Seidal method correct to three decimal places. (8)

ii. Using Jacobi's method, find all the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 4 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$. (8)

13. (a) i. Find the first two derivatives of $x^{1/3}$ at $x = 56$ given the table below:

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

(8)

ii. Apply Romberg's method to evaluate $\int_4^{5.2} \log x dx$, given that

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log_e x$	1.3863	1.4351	1.4816	1.5260	1.5686	1.6094	1.6486

(8)

(Or)

(b) i. Evaluate $\int_{-3}^{3} x^4 dx$ by Trapezoidal and Simpson's $3/8^{th}$ rules with $h = 1$. (8)

ii. Using Simpson's $1/3^{rd}$ rule evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sqrt{\sin(x+y)}dxdy$, taking $h = k = \pi/4$. (8)

14. (a) i. Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by Adams-Bashforth method. (8)

ii. Using Runge-Kutta method of order four, solve $y'' = xy'^2 - y^2$, $y(0) = 1$, $y'(0) = 0$ for $x = 0.2$ correct to 4 decimal places with $h = 0.2$. (8)

(Or)

(b) i. Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = \log(x+y)$, with initial conditions $y = 2$ at $x = 0$, for the range $0 \leq x \leq 0.8$ in steps of 0.2. (8)

ii. Given $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$ and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$, evaluate $y(0.4)$ by Milne's predictor corrector method. (8)

15. (a) i. Solve $y'' - y = x$, $x \in (0,1)$ given $y(0) = y(1) = 0$ using finite differences dividing the interval into 4 equal parts. (8)

ii. Solve $\nabla^2 u = 8x^2y^2$ for square mesh given $u = 0$ on the 4 boundaries dividing the square into 16 sub-squares of length 1 unit each. (8)

(Or)

(b) i. Given $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $u(0,t) = u(5,t) = 0$, $u(x,0) = x^2(25 - x^2)$, find u at $t = 5$ taking $h = 1$ by Schmidt method. (8)

ii. Find the nodal values of the wave equation $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$ given that $u(0,t) = u(5,t) = 0$; $u(x,0) = x^2(5-x)$ and $u_t(x,0) = 0$ taking $h = 1$ and upto one half of the period of vibration. (8)

*****