Reg. No. | | | | | | | | | |

B.E. / B. Tech. (Full Time) END SEMESTER EXAMINATIONS – MAY / JUNE 2012

FOURTH SEMESTER (REGULATION 2008)

31

COMMON TO ALL BRANCHES

MA 9261 – PROBABILITY AND STATISTICS

Time : 3 Hours | Answer ALL Questions | Max. Marks : 100

Part - A (10 × 2 = 20)

1. If a random variable X has a p.d.f $f(x)=\begin{cases} x\, e^{-x}, & x>0 \\ 0, & x\le 0 \end{cases}$, find the moment generating function of X and hence obtain its mean.
2. Let X be a random variable with $E(X)=2$ and $E(X(X-4))=4$. Find $Var(-4X-10)$.
3. Show that the correlation coefficient ρ_{XY} of the random variables X and Y lies between -1 and 1.
4. State the central limit theorem for independent and identically distributed random variables.
5. A laboratory technician as turned 20 times in the performance of a task, getting $\bar{x}=7.9$ and $s=1.2$. Does this constitute evidence against the alternatives that the average time is less than 7.5 minutes. Use $\alpha=0.05$ level of significance.
6. If 12 determinations of the specific heat of iron have a standard deviation of 0.0086, test the null hypothesis that $\sigma=0.010$ for such determinations. Use the alternative hypothesis $\sigma\neq 0.010$ and the level of significance $\alpha=0.01$.
7. State the identity for sum of squares for one-way of analysis of variance.
8. Explain Latin square design.
9. Write the formule for control chart values (Central Line, UCL, and LCL) of a C-chart.
10. To check the strength of carbon steel for use in chair links, yield stress of random sample of 25 pieces was measured, yielding a mean and deviation of 52,800 psi and 4,600 psi respectively. Establish tolerance limits with $\alpha=0.05$, and $p=0.99$.

Part - B (5 × 16 = 80)

11. (i) Ten inspection sample lots of five amplifiers each are drawn from production. The following table lists the average life and range of the power output obtained for each amplifier. Construct $\bar{X}$ - chart, and R-chart. Comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	11.0	12.0	12.8	14.0	13.6	12.8	11.8	12.9	13.0	11.8
R	4	4	6	4	6	5	5	6	4	6

(8 Marks)

(ii) Thirty – five successive samples of 100 castings each taken from a production line, contained respectively 3,3,5,3,5,0,3,2,3,5,6,5,9,1,2,4,5,2,0,10,3,6,3,2,5,6,3,3,2,5,1,0,7,4 and 3 defectives. If the fraction defective is to be maintained at 0.02, construct a p-chart for these data and state whether or not this standard is being met. (8 Marks)

12. (a) (i) Suppose that the random variable X has geometric distribution with $p = 0.5$. Compute (1) $P(X > 2)$ (2) $E(X)$ and (3) $Var(X)$. (5 Marks)

(ii) Let X be a uniform random variable over $(-5, 5)$. Find (1) Cumulative distribution function of X (2) $P(X \leq 0)$ and (3) $P(|X| > 2)$. (6 Marks)

(iii) Suppose the random variable X has the p.d.f

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the p.d.f of the random variable $Y = X^2$ and hence obtain $E(Y)$. (5 Marks)

(OR)

(b) (i) If the p.d.f of the random variable X is $f(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x}, & x > 0 \\ 0, & x \leq 0, \end{cases}$

find (1) cumulative distribution function of X (2) $P(X > 3 \mid X > 2)$ (3) $P\left(\frac{1}{3} < X < \frac{2}{3}\right)$. (6 Marks)

(ii) Determine the binomial distribution for which $E(X) = 4$ and $Var(X) = 3$. Find its moment generating function also. (5 Marks)

(iii) Let X be a uniformly distributed continuous random variable over $[-2, 2]$. Find the probability density function of the random variable $Y = X^3$. (5 Marks)

13. (a) (i) Let the joint p.d.f. of the random variables X and Y be given by

$$f(x, y) = \begin{cases} C\, xy^2, & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(1) Determine the value of C (2) Find the marginal p.d.fs of X and Y (3) Find the conditional p.d.f of X given $Y = y$ (4) Compute $E(XY)$. (8 Marks)

(ii) Suppose the random variable X and Y have the joint p.d.f

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0,\ y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the joint p.d.f of the random variables $U = \frac{X}{Y}$ and $V = Y$ and hence obtain the marginal p.d.fs of U and V. (8 Marks)

(OR)

(b) (i) The joint p.d.f of the random variables X and Y is given as

$$f(x,y)=\begin{cases}2\,e^{-(x+y)}, & 0<y<x<\infty\\ 0, & \text{otherwise}\end{cases}$$

Compute the correlation coefficient ρ_{xy} of X and Y. (8 Marks)

(ii) Suppose the random variables X and Y have the joint p.d.f

$$f(x,y)=\begin{cases}e^{-(x+y)}, & x>0,>0\\ 0, & \text{otherwise}\end{cases}$$

Find (1) $P(X>2, Y<4)$ (2) $P(X>Y)$ (3) $P(X+Y>1)$ (4) Are the random variables X and Y independent? Explain. (8 Marks)

14. (a) (i) Tests of the fidelity and selectively of 190 radio receivers produced the results shown in the following table:

		Fidelity		
		Low	Average	High
Selectivity	Low	6	12	32
	Average	33	61	18
	High	13	15	0

Use the $\alpha = 0.01$ level of significance to test whether there is a relationship (dependence) between fidelity and selectivity. (8 Marks)

(ii) The following are the number of defective pieces turned out by a machine during 24 consecutive shifts:

15,11,17,14, 16,12,19,17,21,15,17,19,21,14,22,16,19,12,16,14,18,17,24 and 13.

Test the null hypothesis of randomness at $\alpha = 0.01$ level of significance. (8 Marks)

(OR)

(b) (i) The following is the distribution of the hoursly number of trucks arriving at a company's warehouse:

Tracks arrival per hour	0	1	2	3	4	5	6	7	8
Frequency	52	151	130	102	45	12	5	1	2

Find the mean of this distribution and using it (rounded to one decimal place) as the parameter λ, fit a Poisson distribution. Test for goodness of fit at the 0.05 level of significance. (8 Marks)

(ii) The following are the number of sales which a sample of nine sales people of industrial chemicals in California and a sample of the six sales people of industrial chemicals in Oregon makes over a certain fixed period of time:

California : 59 68 44 71 63 46 69 54 48

Oregon : 50 36 62 52 70 41

Assume that the populations sampled can be approximated closely with normal distributions having the same variance but unknown. Test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative hypothesis $\mu_1 - \mu_2 \neq 0$ at the 0.01 level of significance. (8 Marks)

15.(a) Four different, through supposedly equivalent, forms of a standardized reading achievement test were given to each of five students, and the following are the scores which they obtained.

	Student 1	Student 2	Student 3	Student 4	Student 5
Form A	75	73	59	69	84
Form B	83	72	56	70	92
Form C	86	61	53	72	88
Form D	73	67	62	79	95

Perform a two-way analysis of variance to test at the level of significance $\alpha = 0.01$ whether it is reasonable to treat the four forms as equivalent. Are the scores of the students significantly difference at $\alpha = 0.01$ level? (16 Marks)

(OR)

(b) A Latin-square design was used to compare the bond strength of gold semiconductor lead wires bonded to the lead terminal by five different methods, A, B, C, D and E. The bonds were made by five difference operators, and the devices were encapsulated using five different plastics with the following results, expressed as pounds of force required to break the bond.

Operator

Plastic	O_1	O_2	O_3	O_4	O_5
P_1	A 3.0	B 2.4	C 1.9	D 2.2	E 1.7
P_2	B 2.1	C 2.7	D 2.3	E 2.5	A 3.1
P_3	C 2.1	D 2.8	E 2.5	A 2.9	B 2.1
P_4	D 2.0	E 2.5	A 3.2	B 2.5	C 2.2
P_5	E 2.1	A 2.6	B 2.4	C 2.4	D 2.1

Analysis these results with $\alpha = 0.01$ level of significance to the mean breaking strengths of the two bonding methods. (16 Marks)

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