B.E / B.Tech (Full Time) DEGREE END SEMESTER EXAMINATIONS, NOV / DEC 2013

Information Technology

12/11/13

Semester 3

(5)

MA 8354 & Probability and Queuing theory

(Regulation 2012)
(Statistical Tables are permitted into the Examination Hall)

Time : 3 Hours | Answer ALL Questions | Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. Suppose that X is a discrete random variable with $E(X)=1$ and $E(X(X-2))=3$. Find $Var(-3X+5)$.

2. Let X be a random variable that has a density function $f_X(x)=k(2x-3x^2), -1<x<0$. Calculate the value of k.

3. Let the conditional probability density function of X, given that $Y=y$ be

$$f_{X|Y}(x|y)=\frac{x+y}{1+y}e^{-x}, 0<x<\infty, 0<y<\infty. \text{ Find } P\{X<1|Y=2\}.$$

4. Find $Cov(X, XY)$.

5. Suppose that on a summer evening, shooting stars are observed at a Poisson rate of one every 12 minutes. What is the probability that 3 shooting stars are observed in 30 minutes?

6. 3 children (denoted by 1,2,3) arranged in a circle play a game of throwing a ball to one another. At each stage the child having the ball is equally likely to throw it into any one of the other two children. Suppose that X_0 denotes the child who had the ball initially and $X_n, n\geq 1$ denotes the child who had the ball after n throws. Find the Transition probability matrix P.

7. Explain Kendall's notation for a single server queuing model.

8. State the steady-state probabilities for $M/M/c:GD/N/\infty$ queuing model.

9. For $M/G/1$ queuing model, obtain the Pollaczeck – Khinchine formula when the service times are deterministic.

10. Distinguish between open and closed queuing networks.

Part – B (5 x 16 = 80 marks)

11. (i) Determine the moment generating function of a Gamma random variable X with parameters (n, λ). Hence obtain the mean and variance of X. **(6)**

(ii) Of police academy applicants, only 25% will pass all the examinations. Suppose that 12 successful candidates are needed. What is the probability that, by examining 20 candidates, the academy finds all of the 12 persons needed? **(5)**

(iii) Let X be a continuous random variable with the probability density function

$$f_X(x)=\frac{2}{x^2}, 1<x<2.$$

Find the cumulative distribution and density functions of $Y=X^2$. **(5)**

12. a) (i) Let X and Y be independent (strictly positive) exponential random variables each with parameter λ. Are the random variables $X+Y$ and $\frac{X}{Y}$ independent? **(8)**

(ii) What is the probability that the average of 150 random points from (0,1) is within 0.02 of the midpoint of the interval? State the result used. **(8)**

OR

b) Let X and Y be random variables having joint density function

$$f_{XY}(x,y)=\frac{6-x-y}{8}\ ,0\le x\le 2\ ,2\le y\le 4\ .$$

Find the correlation coefficient $r(X,Y)$. **(16)**

13. a) For a 3- state Markov chain with state space $S=\{1,2,3\}$ given by the Transition probability matrix:

$$P=\begin{pmatrix}\frac{1}{2} & \frac{1}{4} & \frac{1}{4}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2}\end{pmatrix}$$

Classify the states of the Markov chain and find the steady-state distribution of the Markov chain if it exists. **(16)**

OR

b) (i) State the postulates for a Poisson process. Hence show that the counting process $\{N(t),t\ge 0\}$ follows a Poisson distribution with intensity parameter λ **(10)**

(ii) Given $\{N(t),t\ge 0\}$ to be a Poisson process with intensity parameter λ, for $s<t$, find $P\{N(s)=k|N(t)=n\}$. **(6)**

14. a) (i) Derive the steady-state probabilities for $M/M/c:FCFS/\infty/\infty$ model and hence obtain L_S. **(10)**

(ii) In a 3 server infinite capacity Poisson queueing model with the arrival rate 1/6 per minute and service rate ¼ per minutes, find the average number of customers in the system. **(6)**

OR

b) (i) Derive the steady-state probabilities for $M/M/1:FCFS/N/\infty$ model and hence obtain L_S. **(10)**

(ii) A one- person barber shop has 6 chairs to accommodate people waiting for a hair-cut. Assume that customers who arrive when all the 6 chairs are full, leave without entering the barber shop. Customers arrive at the average rate of 3 per hour and spend an average of 15 minutes in the barber's chair. What is the expected number of customers waiting for a hair cut? **(6)**

15. a) (i) Derive the Pollaczek-Khintchine(P-K) formula for the $M/G/1:GD/\infty/\infty$ model**(10)**

(ii) Suppose a one-person tailor shop is in business of making mens suits. Each suit requires 4 distinct tasks to be performed before it is completed. Assume all 4 tasks must be completed on each suit before another is started. The time to perform each task has an exponential distribution with a mean of 2 hours. If orders for a suit come at an average rate of 5.5 per week, how long can a customers expect to wait to have a suit made?(assume an 8 hour day and 6 day week) **(6)**

OR

b) Consider a Jackson network with three service facilities having the parameter values as shown below:

				p_{ij}		
Facility j	C_j	μ_j	r_j	i=1	i=2	i=3
j=1	1	40	10	0	0.3	0.4
j=2	1	50	15	0.5	0	0.5
j=3	1	30	3	0.3	0.2	0

(i) Find the total arrival rate at each of the facilities.

(ii) Find the expected total number of customers in the system.

(iii) Find the expected total waiting time for a customer. **(16)**

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