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B.E / B.Tech ( Full Time ) DEGREE END SEMESTER EXAMINATIONS, NOV /DEC 2013

Common to ECE/Biomedical

Semester IV

**MA034 / MA504 / MA9263 & PROBABILITY AND RANDOM PROCESSES**

(Regulation 2002 /2004 / 2008)

Time: 3 Hours Answer ALL Questions Max. Marks 100

(Statistical tables are permitted into the exam hall)

**PART-A (10 x 2 = 20 Marks)**

1. Find the probability mass function of a discrete random variable whose CDF is given by

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{6} & 0 \le x \le 2 \\ \frac{1}{2} & 2 \le x \le 4 \\ \frac{5}{8} & 4 \le x < 6 \\ 1 & x \ge 6. \end{cases}$$

2. A random variable X has the density function $f_X(x) = 2e^{-2x}, x > 0$. Using the method of transformations, find the pdf of $Y = \sqrt{X}$ .

3. The joint pdf of the random variables X and Y is defined as $f_{XY}(x,y) = 25e^{-5y}, 0 < x < 0.2, y > 0$. Find the marginal pdfs of X and Y.

4. The joint pdf of two continuous random variables X and Y is given by

$$f_{XY}(x,y) = kx, 0 < y \le x < 1.$$

Determine the values of the constant k.

5. A random process $x(t) = A\cos t + (B+1)\sin t, -\infty < t < \infty$ with A and B to be independent random variables and $E(A) = E(B) = 0, E(A^2) = E(B^2) = 1$. Check if the random process is wide sense stationary.

6. Suppose that customers arrive at a store according to a Poisson process at a rate 10 per hour. Calculate the conditional probability that in 5 hours 20 customers arrived given that in 10 hours 30 customers arrived.

7. An ergodic random process $x(t)$ has the autocorrelation function $R_{xx}(\tau) = 36 + \frac{4}{1+\tau^2}$. Determine the mean value , mean square value and variance of $x(t)$.

8. Give reasons why the function $S_{xx}(w) = \frac{\cos w}{w}$ can or cannot be a power spectral density of a wide- sense stationary random process.

9. Assume that the input $x(t)$ to a linear time-invariant system is white noise. What is the power spectral density of the output process $y(t)$ if the system response:

$$H(w) = \begin{cases} 1 & w_1 < |w| < w_2 \\ 0 & otherwise. \end{cases}$$

10. A random process $x(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \ge 0$. If the autocorrelation function of the process is $R_{xx}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $y(t)$.

**Part – B ( 5 x 16 = 80 marks)**

11. (i) $x(t)$ and $y(t)$ are two jointly wide-sense stationary processes. If $Z(t) = x(t) + y(t)$ is the input to a linear system with impulse response $h(t)$, determine: the autocorrelation of $Z(t)$; the power spectral density of $Z(t)$; the cross-power spectral density $S_{ZV}(w)$ of the input process $Z(t)$ and the output process $V(t)$ and ,the power spectral density of the output process $V(t)$. **(10)**

(ii) Given the auto correlation function $R_{xx}(\tau) = e^{-a|\tau|}$, $a$ is a real positive constant, of a random process $x(t)$, if the input $x(t)$ is applied to a linear time invariant system with impulse response $h(t) = e^{-bt}u(t), b$ is a real positive constant. Find the auto correlation function of the output process of the system. **(6)**

12. a) (i) A certain student is known to be late to the signals and systems class 30% of the time. If the class meets four times a week, find
1. The probability that the student is late for at least three classes in a given week;
2. The probability that the student will not be late at all during a given week. **(8)**
(ii) Derive the moment generating function of a Gamma random variable X. Also, find the mean and variance of the random variable X. **(8)**

OR

b) (i) A student is planning to take the scholastic aptitude test exam to gain admission to a top college. She hopes to keep taking the exam until she gets a score of at least 2000 and then she will stop. Her score in any of the exams is uniformly distributed between 800 and 2200, and her score in one exam is independent of her score in any other exam.
1. What is the probability that she reaches her goal of scoring at least 2000 points in any exam?
2. What is the pmf of the number of times she will take the exam before reaching

her goal?
3. What is the expected number of times she will take the exam? **(8)**
(ii) Derive the moment generating function of a Normal random variable. Also, find the mean and variance of X. **(8)**

13. a) Given $f_{XY}(x,y)=\frac{1}{3}(x+y), 0\le x\le 1, 0\le y\le 2$, find the correlation coefficient $r(X,Y)$. **(16)**

**OR**

b) (i) If $X\sim N(0,1)$ and $Y\sim N(0,1)$ are normally distributed, independent random variables, find the joint pdf of $R=\sqrt{X^2+Y^2}$ and $\theta=\tan^{-1}\left(\frac{Y}{X}\right)$. Show that $R$ and $\theta$ are independent. **(10)**
(ii) What is the probability that the average of 150 random points from (0,1) is within 0.02 of the midpoint of the interval? **(6)**

14. a) Define a Poisson process. Also, state the postulates of a Poisson process. Hence, obtain $P_n(t)=P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}, n=0,1,2...$ **(16)**

**OR**

b) (i) Consider the random process $y(t)=(-1)^{x(t)}$, where $x(t)$ is a Poisson process with rate $\lambda$. Verify whether the process $y(t)$ is wide-sense stationary. **(8)**
(ii) Prove that the random process $x(t)=\cos(t+\Phi)$, $\Phi$ is a random variable with $f_\Phi(x)=\frac{1}{2\pi}, 0<x<2\pi$ is 1.First order stationary, 2. Second- order stationary **(8)**

15. a) (i) Consider a random process $x(t)=A_0\cos(tw_0+\theta)$, $y(t)=A_1\cos(w_1t+\Phi)$, where $A_0.A_1,w_0,w_1$ are constants and the random variables $\Phi$ and $\theta$ are independent and uniformly distributed over $(-\pi,\pi)$. Find
1. $R_{xy}(\tau)$ if $\theta\ne\Phi$ 2. $R_{xy}(\tau)$ if $\theta=\Phi$ **(10)**
(ii) The auto correlation function of a random process is given by $R_{xx}(\tau)=a^2e^{-2k|\tau|}$, $a$ is a constant. Find $S_{xx}(w)$. **(6)**

**OR**

b) (i) Find the average power of a random process $x(t)$ if its power spectral density is given by $S_{xx}(w)=\frac{10w^2+35}{(w^2+4)(w^2+9)}$ **(8)**
(ii) Two zero-mean independent wide sense stationary random processes of $x(t)$ and $y(t)$ have the power spectral densities: $S_{xx}(w)=\frac{4}{w^2+4}$ and $S_{yy}(w)=\frac{w^2}{w^2+4}$ respectively. Two new random process $V(t)$ and $W(t)$ are defined as: $V(t)=x(t)+y(t), W(t)=x(t)-y(t)$ respectively. Determine the cross- power spectral density $S_{VW}(w)$ and hence the cross correlation function $R_{VW}(\tau)$. **(8)**

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