25/10/13

(12)

Reg. No.								

B.E. / B.Tech. DEGREE END SEMESTER EXAMINATIONS, NOV / DEC 2013

FOURTH SEMESTER

MA 039 / MA 503 / MA 9261 – PROBABILITY AND STATISTICS

(Common to Industrial, Manufacturing, Geo informatics, Bio Technology, Textile and Apparel)

TIME : 3hrs **MAX.MARKS** : 100

Answer ALL Questions

Part – A (10 x 2 = 20 Marks)

1. Suppose the discrete random variable X has the probability mass function (p.m.f.)

$$P(X=x)=\begin{cases} Cx & , \quad x=1,2,\ldots,10 \\ 0 & , \quad \text{elsewhere.} \end{cases}$$

Find (i) value of C (ii) $P\left(\frac{1}{2}<X<\frac{5}{2}\right)$.

2. A continuous random variable X has the probability density function (p.d.f.) $f(x)=\frac{1}{2}e^{-|x|}, -\infty<x<\infty$. Find $E(X)$.

3. If X = Y then show that $Cov(X,Y)=Var(X)=Var(Y)$.

4. State the central limit theorem for independent and identically distributed random variables.

5. If $n=40$, $\bar{x}=14$, $\mu=10$ and $\sigma=12$, is there any significant difference between $\bar{x}$ and μ at 5% level of significance?

6. If 12 determination of the specific heat of iron have a standard deviation of s = 0.0086, test the null hypothesis that $\sigma=0.010$ for such determinations. Use the alternative hypothesis $\sigma\neq 0.010$ and the level of significance $\alpha=0.01$.

7. Define (i) Degrees of freedom (ii) Mean square.

8. Why a 2 x 2 Latin square is not possible? Explain.

9. Write the formulae for control chart values (Central line, UCL and LCL) of a p - chart.

10. Control charts for $\bar{X}$ and R to be set up for an important quality characteristic. For the sample size $n=4$, $\bar{x}$ and $\bar{r}$ are computed for each of 30 prelimmary samples. The summary data are $\sum_{i=1}^{30}\bar{x}_i=712.5$, $\sum_{i=1}^{30}\bar{r}_i=143.7$. Find the trail control limits for $\bar{X}$ and R.

Part – B **(5 x 16 = 80 Marks)**

11. i) The target till weight for a box of cereal is 350g. Each day a sample of 300 boxes is taken and the number that are underweight is counted. The number of underweight boxes for each of the last 25 day is as follows?

23	12	19	19	20	19	21	27	26	23	26	22	25
30	30	22	25	27	29	39	43	41	39	29	.	

1) Compute the UCL and LCL for a p - chart.
2) Is the process in control? If not when it is first detected to be out of control? **(8)**

ii) Each hour, a 10 m^2 section of fabric is inspected for flaws. The numbers of flaws observed for the last 20 hours are as follows?

38	35	35	49	33	48	40	47	45	46
41	53	36	41	51	63	35	58	55	57 .

1) Compute the upper and lower control limits for a C - chart.
2) Is the process in control? Explain. **(8)**

12. (a) (i) The continuous random variable X has the cumulative distribution function (CDF)

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ x^2 & , \quad 0 \le x < 1 \\ 1 & , \quad x \ge 1 \end{cases}.$$

Compute 1) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ 2) $P\left(X > \frac{1}{2}\right)$ 3) $E(X)$. **(6)**

(ii) Determine the binomial distribution for which $E(X) = 4$ and $Var(X) = 3$. Find its moment generating function (MGF) also. **(5)**

(iii) Let X have the p.d.f. $f(x) = \begin{cases} 1 & , \quad 0 < x < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$.

If the random variables $Y = -2 \log X$, find the p.d.f. of Y and $E(Y)$. **(5)**

(OR)

(b) (i) Let X be a uniform random variable over $(-2, 2)$. Find 1) Cumulative distribution function of X, 2) $P(X \le 1)$ 3) $P(|X| > 1)$.

(6)

(ii) If the p.d.f. of the random variable X is

$$f(x) = \begin{cases} \frac{1}{5} e^{-\frac{1}{5}x} & , \quad x > 0 \\ 0 & , \quad x \le 0 \end{cases}$$

find 1) $P(X > 5 / X > 4)$ 2) $P\left(\frac{2}{3} < X < 1\right)$ 3) $E(X)$. **(5)**

(iii) Let X be the discrete random variables with the p.m.f.

$$P(X = x) = \begin{cases} \frac{1}{K} & , \quad x = 1, 2, 3, \ldots, K \\ 0 & , \quad \text{otherwise} \end{cases}.$$

Find the moment generating function of X and hence obtain $E(X)$. **(5)**

13. (a) (i) The joint p.d.f. of the continuous random variables X and Y is given as

$$f(x,y)=\begin{cases}\frac{1}{6}(x+y) & , \ 1\le x\le 2,\ 4\le y\le 5\\ 0 & , \ \text{otherwise}\end{cases}$$

Find 1)) $P\left(1\le X\le \frac{3}{2}, \frac{9}{2}\le Y\le 5\right)$, 2) The marginal p.d.fs of X and Y,

3) The conditional p.d.f. of Y given $X = 1.2$,

4) Are the Random Variables X and Y independent? Explain. **(8)**

(ii) Let X and Y have the joint p.d.f.

$$f(x,y)=\begin{cases}10xy^2 & , \ 0<x<y<1\\ 0 & , \ \text{otherwise}\end{cases}$$

. Find the joint p.d.f. of the random variables $U=\frac{X}{Y}$ and $V = Y$ and hence obtain the marginal p.d.fs of U and V . **(8)**

(OR)

(b) (i) Suppose the random variables X and Y have joint p.d.f.

$$f(x,y)=\begin{cases}x+y & , \ 0<x<1,\ 0<y<1\\ 0 & , \ \text{elsewhere}\end{cases}$$

Find 1) The marginal p.d.fs of X and Y. 2) $P\left(X<\frac{1}{2}\right)$

3) $P(X+Y<1)$ 4) $E(XY)$. **(8)**

(ii) The joint p.m.f. of the discrete random variables X and Y is given as

X \ Y	0	1	2
0	0.05	0.10	0.20
1	0.05	0.15	0.05
2	0.25	0.10	0.05

Find Cov (X,Y) and the correlation coefficient ρ_{XY} of X and Y. **(8)**

14. (a) (i) The following random samples are measurement of heat-producing capacity (*in millions of calories per ton*) of specimens of coal from two mines:

Mine I :	8260	8130	8350	8070	8340	
Mine II :	7950	7890	7900	8140	7920	7840

Use the $\alpha = 0.01$ level of significance to test whether the difference between the means of these two samples is significant? **(8)**

(ii) Suppose that 150 randomly selected third-year medical students were rated *according to their success in medical school and their ability in mathematics.* With respect to each of these characteristics the students were rated as low, average or high. The number of students in each category is given in the following contingency table:

	Ability in Mathematics		
Success in Medical School	Low	Average	High
Low	14	8	5
Average	12	51	11
High	7	24	18

Use the $\alpha = 0.05$ level of significance to test whether the success in medical school is related to ability in mathematics? **(8)**

(OR)

(b) (i) The following is a record of the two-alarm fires that occurred in a certain town. On the basis of these data, would it be reasonable to assume that the number of two-alarm fires in any given day in this town is a Poisson random variable? Test at the 0.05 level of significance.

Number of Fires	0	1	2	3	4	5	6	7
Number of Days	151	118	77	19	0	0	0	0

(8)

(ii) The following are the scores which random samples of students from two minority groups obtained on a current events test:

Group 1:	73	82	39	68	91	75	89	67	50	86	57	65	70
Group 2:	51	42	36	53	88	59	49	66	25	64	18	76	74

Use the U-test at the 0.05 level of significance to test whether or not students from two minority groups can be expected to score equally well on the test. **(8)**

15. (a) An experiment was performed to judge the effect of 4 different fuels and 3 different types of launchers on the range of a certain rocket. Test on the basis of the following ranges, in miles, whether there is a significant effect due to differences in fuels and whether there is a significant effect due to differences in launchers. Use $\alpha = 0.05$ level of significance.

Launcher \ Fuel	1	2	3	4
I	45.9	57.6	52.2	41.7
II	46.0	51.0	50.1	38.8
III	45.7	56.9	55.3	48.1

(16)

(OR)

(b) The following data relate to the results of a Latin square experiment on four varieties of paddy A, B, C and D:

18 A	21 C	25 D	11 B
22 D	12 B	15 A	19 C
15 B	20 A	23 C	24 D
22 C	21 D	10 B	17 A

Analyze the results and offer your comments at 5% level of significance. **(16)**

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