

Time : 3 hours

Max Marks: 100

PART – A (10 x 2 = 20 marks)

Answer ALL the Questions

1. Prove that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ using induction.
2. Define finite automata and NFA. Give an example for each.
3. Write regular expressions for the following :
 - i) Set of all strings with 0's and 1's beginning with 0 and ending with 1.
 - ii) Set of all strings having even number of 1's
4. Prove that the regular sets are closed under intersection.
5. Draw the parse tree for the statement $A := B + C * A$ using the following grammar:
 $\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle := \langle \text{expr} \rangle$
 $\langle \text{id} \rangle \rightarrow A \mid B \mid C$
 $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \mid \langle \text{term} \rangle$
 $\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle \mid \langle \text{factor} \rangle$
 $\langle \text{factor} \rangle \rightarrow (\langle \text{expr} \rangle) \mid \langle \text{id} \rangle$
6. Define Push Down Automata. State its applications.
7. Eliminate the ϵ -productions and null productions for the following grammar.
 $S \rightarrow ABaC$
 $A \rightarrow BC$
 $B \rightarrow b \mid \epsilon$
 $C \rightarrow D \mid \epsilon$
 $D \rightarrow d$
8. Design a TM to accept the string $\{a, b\}^* aba$.
9. Eliminate left recursion from the following grammar.
 $S \rightarrow Aa/b$
 $A \rightarrow Ac \mid Sd \mid \epsilon$
10. Define Top-down and Bottom-up parsing and write the type of grammars used for them.

PART – B (5 x 16 = 80 marks)

11. i) Prove that $\sqrt{2}$ is an irrational number. Provide your proof using “proof by contradiction”. (8)

ii) Let L be a set accepted by a finite automation. Prove that there exists a DFA that accepts L . (8)

12. a) (i) State and Prove the pumping Lemma. Using this Lemma, prove that the set $L = \{0^21^2\}$ is not regular. (8)

(ii) Let r be a regular expression. Prove that there exists a NFA – with epsilon that accepts $L(r)$. (8)

{or}

b) Explain the procedure used for minimizing the DFA. Using this procedure, minimize the DFA given in Figure 1. (16)

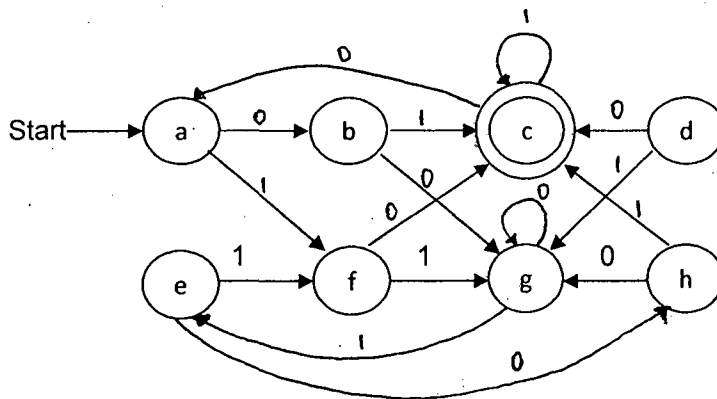


Figure 1 – Finite Automata

13. a) Write the procedure to convert a CFG to Greibach Normal Form. Using this procedure, convert the following grammar to GNF. (16)

$G = (\{A_1, A_2, A_3\}, \{a, b\}, P, A_1)$ where P consists of the following production rules.

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 / b$

$A_3 \rightarrow A_1 A_2 / a$

{or}

b) Design a PDA which accepts $L = \{wcw^r / w \in \{a, b\}^*\}$ i.e., the string of palindromes over $\{a, b\}$. Write Instantaneous Description for the valid string “MALAYALAM”. Write the context free grammar corresponding to this PDA. (16)

14. a) (i) Define CFG. Explain the following with examples (8)
1. Left Most Derivation.
 2. Right Most Derivation.
 3. Ambiguous Grammar
 4. Left Recursive grammar.

(ii) Consider the grammar. (8)

$\langle S \rangle \rightarrow \langle NP \rangle \langle VP \rangle$

$\langle VP \rangle \rightarrow \langle V \rangle \langle NP \rangle$

$\langle NP \rangle \rightarrow \langle NAME \rangle$

$\langle NP \rangle \rightarrow \langle ART \rangle \langle N \rangle$

$\langle ART \rangle \rightarrow a / the$

$\langle V \rangle \rightarrow ate / saw$

$\langle N \rangle \rightarrow cat / mouse$

$\langle NAME \rangle \rightarrow Ram / Kumar$

Draw parse trees to represent the sentences

- i) Ram saw the mouse.
- ii) The cat ate the mouse

Provide a Left most derivation to recognize the sentence, "Ram saw the mouse".

(or)

- b) (i) Define Turing Machine and explain it. (8)
- ii) Design a Turing Machine to check whether a given string is a palindrome or not. (8)

15. a) i) Explain the predictive parsing algorithm with an example. (8)

ii) Find the FIRST() and FOLLOW() for the following grammar. (8)

$E \rightarrow TE'$

$E' \rightarrow +TE' / \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' / \epsilon$

$F \rightarrow (E) / id$

(or)

- b) i) Define LL(1) grammar. Give an example for LL(1) grammar. (3)
- ii) Define LR grammar and give an example for LR grammar. (3)
- iii) Explain the Bottom-up parsing using a suitable example. (10)